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How do exemplary Chinese and U.S. mathematics teachers view instructional coherence?

Jinfa Cai · Meixia Ding · Tao Wang

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Abstract The purpose of this cross-national study is to understand teachers' views about the meaning of instructional coherence and the ways to achieve instructional coherence. With respect to the meaning of instructional coherence, whereas the majority of U.S. teachers paid attention to connections between teaching activities, lessons, or topics, the majority of Chinese teachers emphasized the interconnected nature of mathematical knowledge beyond the teaching flow. U.S. teachers expressed their views about ways to achieve instructional coherence through managing a complete lesson structure. In contrast, Chinese teachers emphasized pre-design of teaching sequences, transitional language and questioning based on the study of textbooks and students beforehand. Moreover, they emphasized addressing student thinking and dealing with emerging events in order to achieve "real" coherence. The findings of the study contribute to our understanding about the meaning of instructional coherence and ways to achieve instructional coherence in different cultural contexts.

Keywords Instructional coherence · Discourse · Cross-cultural study · China · USA · Learning · Effective teaching · Teachers' views

1 Introduction

Instructional coherence has been identified as an important feature of effective instruction in mathematics classrooms (Cai, Kaiser, Perry, & Wong, 2009; Stigler & Hiebert, 1999). In particular, the direct relationship between East Asian students' exposure to coherent instruction and their performance in mathematics (Cai et al., 2009; Hiebert et al., 2003; Stigler & Hiebert, 1999) has led to increasing recognition of the significance of instructional coherence for students' mathematical learning (Leung, 1995; Shimizu, 2007; Wang & Murphy, 2004). However, there are at least two unresolved issues in the existing literature. First, there

J. Cai (✉)
University of Delaware, Newark, DE, USA
e-mail: jcai@math.udel.edu

M. Ding
Temple University, Philadelphia, PA, USA

T. Wang
The University of Tulsa, Tulsa, OK, USA

is not a clear consensus on the definition of instructional coherence in mathematics classrooms due to its complex nature. This conceptual issue becomes even more complicated when mathematical instruction is seen as a cultural activity (Cai, 2004). Second, although researchers have reported differences in instructional coherence between U.S. and East Asian mathematics classrooms (Leung, 1995; Shimizu, 2007; Wang & Murphy, 2004), we know little about how mathematics teachers themselves in different countries view instructional coherence. In fact, teachers from different countries hold different views of effective mathematics teaching which affect their instructional practices in classrooms (Cai et al., 2009). Therefore, there is both theoretical and practical significance in studying the views of teachers from different countries about instructional coherence, a critical construct in effective mathematical teaching.

The purpose of this study is to examine how exemplary U.S. and Chinese mathematics teachers view instructional coherence in the classroom. In particular, this study has been designed to answer two research questions: (1) How do exemplary mathematics teachers view the meaning of instructional coherence? and (2) How do exemplary mathematics teachers view possible ways to achieve instructional coherence? We used a cross-national comparative approach to answer these two questions. This study is significant because it can inform our understanding of what instructional coherence may entail and how teachers may achieve instructional coherence.

2 Theoretical basis

2.1 What is coherence?

Coherence has long been a construct in research related to discourse, and recently it has become a construct in educational research. Given the fact that instruction involves classroom discourse, it is helpful to review how coherence has been studied in both areas.

Discourse coherence reflects the degree of meaning related to topics, which affects readers' or listeners' understanding (Dore, 1985). A coherent story, for example, with thematically related events is easier for the reader or listener to comprehend. However, discourse coherence is not a flat, linear entity because the organization of discourse is hierarchical and interwoven. According to van Dijk (1997), there are both micro and macro levels of coherence. At the micro level, "connections between propositions in composite sentences and successive sentences" are present (van Dijk, 1997, p. 4). At the macro level, coherence refers to the global meaning or theme of the discourse. A similar hierarchical conception of coherence is found in curriculum research. For example, Schmidt, Wang, and McKnight (2005) looked at curriculum content and defined coherence as a sequence of topics and performances consistent with the logical and, if appropriate, hierarchical nature of the disciplinary content from which the subject matter derives. Curriculum coherence also involves different levels, such as within a particular grade or across grades.

Regardless of the level of coherence, a coherent text (what is heard or read) may be processed in different ways, resulting in different mental models such as *textbase* or *situation models* (van Dijk & Kintsch, 1983). Textbase refers to coherent representations of the meaning of the text itself whereas the situation model refers to one's integration of the text content into an existing knowledge system, which is at a higher level than a textbase (Kintsch, 1986). A coherent text with full information may contribute to the formation of a textbase, which may improve one's ability to remember a text. However, it may also reduce one's active processing which may hinder the formation of a situation model. In contrast, a text with some missing information may facilitate inference-making and contribute to the formation of situation models, which are the keys to solving problems and learning from a text (Mannes & Kintsch, 1987; McNamara, Kintsch, Songer, & Kintsch, 1996). In that sense, coherence that results in a textbase may be viewed as less spontaneous and dynamic and more superficial than coherence that results in situation models.

2.2 What is coherence in mathematics instruction?

Recently, the idea of coherence has attracted attention in the field of mathematics education. Many researchers have adopted frameworks and research methods from other disciplines to study instructional coherence in mathematics classrooms, with a focus on lesson structures and events within and across lessons. For example, international studies have revealed that instructional coherence is a distinguishing characteristic of mathematics teaching in China (and other Asian countries) in comparison with the U.S. (Cai, 2005; Fernandez, Yoshida, & Stigler, 1992; Leung, 1995; Stevenson & Stigler, 1992; Stigler & Hiebert, 1999). One feature of instructional coherence is connectedness to a topic. For instance, Stigler and Perry (1990) chunked classroom flow into 5-min segments and found that 55 % of the segments in Taipei (China) classrooms focused on only one topic, in contrast to 17 % of the segments in Chicago classrooms. Focusing on one topic, they argued, made the Chinese lessons thematically more coherent. Other researchers found that some Chinese teachers (e.g., Leung, 1995; Wang & Murphy, 2004) and Japanese teachers (Segiguchi, 2006; Stigler & Hiebert, 1999) devoted an entire 40-min class to only one mathematics problem. For example, Leung (1995) observed 36 elementary lessons in different schools in Beijing and found that all the lessons strictly followed a clear structure that promotes coherence. Likewise, Wang and Murphy (2004) found that all of the teaching activities in a Shanghai lesson were related to one mathematical topic. Chen and Li (2010) further revealed that content connections could be found not only within single lessons but also across lessons in China.

Another feature of instructional coherence identified in Chinese classrooms is the use of transitional language. Wang and Murphy (2004) found that the Chinese teacher in their study often employed transitional language to explicitly to link two adjacent lesson segments [e.g., “Just now we (did something)… Now it is time (to do other things)…”]. These transitions were similar to Japanese teachers’ *matome* that summarized the main points of learning activities and/or set the stage for introducing new activities (Shimizu, 2007). Such transitional language is believed to smooth the teaching flow and facilitate connections between teaching activities.

2.3 Why study teachers’ views about instructional coherence?

The studies mentioned above have revealed some important differences in classroom enactment of instructional coherence which reflect distinctive, cultural beliefs about teaching (Bruner, 1996; Cai & Wang, 2010; Cai et al., 2009; Nasir, Hand, & Taylor, 2008; Stigler & Hiebert, 1999). Behind these observed instructional features, what are the underlying views that guide teachers’ design of coherent lessons? What is the essence of instructional coherence in teachers’ eyes? How do teachers actually view the possible ways to achieve instructional coherence? So far, there have been no studies that have explored these research questions. In addition, while studying teachers’ beliefs is a significant line of research in mathematics education (e.g., Philipp, 2007), most researchers have inferred teachers’ beliefs through interpreting observable instructional behaviours (Wilson, Cooney, & Stinson, 2005;

Stigler & Hiebert, 1999). Little has been done to reveal teachers' beliefs from their own perspectives, especially with teachers from different cultural backgrounds. In this study we are primarily interested in teachers' perspectives about instructional coherence and possible ways to achieve instructional coherence.

3 Method

3.1 Subjects

A total of 20 Chinese teachers and 16 U.S. teachers participated in the study. The Chinese teachers were selected from 13 different provinces because they were considered excellent teachers; 55 % had received top prizes in national mathematics teaching competitions, and 95 % had earned local (e.g., province-wide) teaching awards. The Chinese teachers were participants in a workshop conducted by one of the authors. The survey instrument used to gather data for this study was administered by the workshop organizers about 3 weeks before the actual workshop. At that point, the teachers had not met the author. After the Chinese sample was obtained, 16 comparable U.S. teachers were recruited from 14 different states through an email discussion list of presidential awardees for teaching excellence in mathematics. Thus, the 20 Chinese teachers and 16 U.S. teachers were all considered exemplary teachers who had earned local and/or national reputations in mathematics teaching. Table 1 shows their background information. Both samples had similar proportions of male and female teachers. All of the teachers had rich teaching experience. Seventy percent of the Chinese teachers and nearly 60 % of the U.S. teachers had over 20 years of teaching experience.

Despite these similarities, there were two differences. First, although all the teachers specialized in teaching mathematics, the Chinese teachers taught at the elementary level while the majority of the U.S. teachers taught at the middle and/or high school levels. This was intentional in our sampling because we wanted to have teachers who specialized in

Table 1 Background information of the U.S. and Chinese teacher samples

		U.S. teacher		Chinese teacher	
		Number	Percentage	Number	Percentage
Gender	Male	7	44 %	9	45 %
	Female	9	56 %	11	55 %
Years of teaching	<10	1[a]	6 %	0	0
	10–19	6	38 %	6	30 %
	≥ 20	9	57 %	14	70 %
Awards	National	16	100 %	11	55 %
	Local	10	63 %	19	95 %
Grade level	Elementary	1	6 %	20	100 %
	Middle	10	63 %	0	0
	High	5	31 %	0	0
Highest Degree	BA	0	0	19	95 %
	MA	16	100 %	1	5 %

[a] This teacher had 9 years of teaching experience

teaching mathematics and who would have a greater potential for insightful responses about instructional coherence in mathematics. In China, elementary teachers move from grade to grade with their students and specialize in teaching mathematics, whereas in the U.S., those who specialize in mathematics teaching are generally secondary, not elementary, teachers. As instructional coherence is a feature of effective mathematics teaching at all grade levels, the grade level difference between the two samples was unlikely to be a concern when understanding teachers' views. The second difference between the two groups was that the U.S. teachers held higher academic degrees than the Chinese teachers. All of the U.S. teachers held a master's degree, compared with only one of the Chinese teachers. All of the other Chinese teachers held bachelor's degrees.

3.2 Survey questions

There are various methods to study instructional coherence, such as Likert-scale questionnaire surveys and lesson observations. Since the goal of this study was to understand teachers' views of instructional coherence from *their own* perspectives, we decided to design open-ended survey questions to elicit their free responses in the context of instructional coherence in mathematics teaching. Each of the teachers was given a set of questions and asked to provide written responses. This paper is based on responses to the following three questions:

1) When people say a lesson is very coherent, what does the word "coherent" mean to you? What are the characteristics of a coherent lesson?
2) If you mentor a new teacher, how would you guide the new teacher to achieve coherence in her or his teaching?
3) Some people say that a coherent lesson can foster students' learning. Do you agree with this statement? Why?

Whereas question 1 (Q1) asked about the teachers' overall understanding of instructional coherence, question 2 (Q2) was intended to elicit the teachers' views regarding how instructional coherence may be achieved. Question 3 (Q3) ostensibly focused teachers' attention on the effect of instructional coherence on student learning, but was actually designed to further probe teachers' views about the nature of instructional coherence. Although these three questions had different foci, they were designed to triangulate with each other.

The questions were phrased in ways to elicit relatively objective and rich responses. For example, we used the words "when people say" and "some people say" rather than "research says" so as not to bias teachers. We also suggested a situation ("if you mentor a new teacher") that could help teachers recall personal experiences, thus affording rich responses about how to reach instructional coherence.

3.3 Translation equivalence

To ensure the equivalence of the translations of the instruments, we employed the process of English back-translation conducted by two people who are literate in both Chinese and English. The first person translated the original survey questions from English into Chinese. The second person then translated them back into English. This final translation was then compared to the original to ensure equivalence and consistency. The Chinese version was reviewed by a Chinese mathematics educator and a teacher. The English version was reviewed by two U.S. mathematics teachers. The reviews indicated that both Chinese and

U.S. teachers clearly interpreted the three survey questions as focusing on the same issue of instructional coherence. The final versions of the instrument were edited and revised slightly to reflect the feedback from these reviewers. We paid special attention to the word "coherence" because of its importance in this study. The Chinese characters 连贯 were chosen as a translation for "coherence." Our own experience, the reviewers' comments, and actual data confirmed the linguistic equivalence between "coherence" and "连贯."

3.4 Procedures and data analysis

Each teacher was provided with the same questions with an expectation of free responses and no time restriction. Once the data were collected, we first read through individual teacher responses across the three questions to obtain a general sense of teachers' views. We then analyzed teacher responses to each question using a constant comparison method (Gay & Airasian, 2000). We began by generating a list of key terms based on each teacher's responses. For example, when coding the response of Chinese teacher number 1 (CH1) to Q1 regarding what a coherent lesson is, we obtained codes such as "connection among knowledge pieces," "communication and discussion from the surface to depth," "clear teaching goal," "designing questions from easy to hard," and "dealing with emerging events." When coding another teacher's response, we added new codes if the existing ones did not apply.

After coding each individual question, we triangulated teachers' responses across questions. Consistencies and inconsistencies within teachers' responses were recorded. Most of the time, teachers' responses were consistent. For example, a teacher might view having a clear goal as one of the features of a coherent lesson (Q1) and also mention that he or she would help novice teachers to set up teaching goals (Q2). Similarly, a teacher might express that coherence should not be equated with smoothness of teaching (Q1) and then elaborate on this view by discussing the effects of different types of instructional coherence on student learning (Q3).

After developing the codes, we sorted and combined them into two broad categories: statements about mathematical content connections and statements about the interactive instructional process. For the mathematical content connections, we further classified the teacher responses into macro and micro levels. For the interactive instructional process, we classified the teacher responses into planning and teaching stages. In addition, we analyzed the teachers' views about the impact of instructional coherence on student learning. After analyzing the views of the teachers in each country, we compared Chinese and U.S. teachers' responses and identified the common and different views. To ensure reliability, two independent coders went through the complete data set using similar procedures. The two raters achieved 92 % agreement. The disagreements arose in the classification of three kinds of responses (setting up teaching goals, connecting to student prior knowledge, and challenging student thinking) and were resolved through discussion.

4 Results

We report the teachers' views about instructional coherence using the two primary categories: the mathematical content connections and the interactive instructional process. Although these were often interrelated in teachers' responses, we report them separately for convenience of presentation. Afterwards, we report the teachers' views about conditions of instructional coherence for fostering student learning.

4.1 Mathematical content connections

Both the U.S. and Chinese teachers discussed mathematical content at the micro and macro levels, arguing that content within and between lessons, respectively, should be connected. In our analysis, therefore, the macro level refers to the broader curriculum and the micro level refers to content within a lesson. Despite the similarities between the teachers' views, there were subtle differences in emphasis. Although the majority of U.S. teachers kept their attention on teaching activities, lessons, or topics, the majority of Chinese teachers emphasized the interconnected nature of mathematical knowledge per se (see Table 2).

4.1.1 Macro level

Nine U.S. teachers (56 %), but only five Chinese teachers (25 %), discussed the connections between lessons/topics (see Table 2). The U.S. teachers frequently referred to external sources, such as "content standards" or "scope and sequence." US5 stated, "Lessons are organized and interconnected with a consistent theme which students can clearly see the topic being discussed" and "curricula tied to strong content standards that follow a set of progressions that build on each other." US9 echoed this idea, "If a lesson is chosen to be taught because of its location in the scope and sequence of a certain grade, and followed up by the next sequential lesson, teaching will be cohesive."

The five Chinese teachers who discussed connections in mathematical content at a macro level appeared to share the U.S. teachers' views. For example, CH15 mentioned that coherence meant "to connect each lesson and each unit." However, Chinese teachers' discussions considered more of the coherent nature of the mathematical knowledge system itself.

> Instructional coherence demands appropriate organization of important mathematical ideas… As such, a teacher should try best to organize the mathematical knowledge pieces to make them as a whole and to connect prior and new knowledge pieces. … Finally the connected knowledge shapes an integrated knowledge system (CH5).

4.1.2 Micro level

At the micro level, 14 U.S. teachers (87.5 %) and 17 Chinese teachers (85 %) discussed what connected mathematical content might look like within a lesson (see Table 2). Among the U.S. teachers, 10 of them focused their attention on the "flow" within a lesson. For example, they argued that a coherent lesson should "flow logically from one component of the lesson

Table 2 Mathematical content connections in U.S. and Chinese teachers' views

	Macro		Micro	
	U.S.	CH	U.S.	CH
Connection of activities/ lessons/topics	3, 5, 6, 8, 9, 10, 12, 13	15	1, 3, 4, 5, 6, 7, 8, 11, 12, 13	4, 12, 13, 16
Connection of knowledge pieces	16	2, 5, 10, 15, 19	2, 14, 15, 16	1, 3, 5, 6, 7, 8, 9, 10, 14, 15, 18, 19, 20

to the next" (US1), "have a very close connection between the instructional content and activities done in class" (US5), or "flow easily from one thing to the next very clearly and with purpose" (US6). These views are consistent with observed features reported in the literature such as connected teaching segments and activities (Chen & Li, 2010; Leung, 1995). To elaborate what is meant by flowing from one component/activity/thing to the next, half of the U.S. teachers (US1, US4, US6, US7, US8, US11, US12, and US14) described what should be done in the *beginning, middle, and end* (BME) of a lesson. US11 explained, "A coherent lesson should flow and make sense for the outcome that is expected. The lesson should have a sequence of beginning, middle, and end to it and the students need to see that flow."

Seventeen Chinese teachers acknowledged the flow within a lesson. Four of them (CH4, CH12, CH13, and CH16) mentioned the natural transition among teaching activities and phases. The remaining 13 teachers specifically emphasized the importance of the underlying interconnection of mathematical ideas, which was similar to the pattern observed at the macro level (see Table 2). In an interesting contrast, CH8 noted a popular view of "coherence" as containing well-organized and connected teaching phases, such as an attractive beginning, an exciting middle, and a provoking ending (similar to the U.S. teachers' BME). However, for CH8, coherence needed to go beyond smoothness of teaching flow and "should mean more on the connectedness of knowledge and the systematic nature of students' thinking processes." In alignment with this emphasis, some Chinese teachers expressed their views that experienced teachers should help new teachers first "clarify the mathematical logic behind teaching contents" so that they could "understand how the mathematical concept is originated and developed, and understand the connections among knowledge pieces, thus, grasping the mathematical essence and meaning of the teaching content" (CH9).

4.2 Interactive instructional process

In addition to the mathematical content connectedness as one of the main features of instructional coherence, both groups of teachers emphasized coherence from the perspective of an interactive instructional process. In particular, they discussed processes at the planning stage and the teaching stage about instructional coherence.

4.2.1 Planning stage

When addressing the planning of lessons in their responses, the two groups of teachers showed differences in terms of understanding students and textbooks even though they both agreed with the importance of setting up clear teaching objectives. Table 3 summarizes the Chinese and U.S. teachers' views.

Understanding students Seventeen Chinese teachers (85 %) mentioned that they would guide new teachers to study students, including their prior knowledge, thinking, and learning

Table 3 Teachers' different emphases during the planning stage

	U.S.	CH
Understanding students	1, 2, 3, 4, 5, 6, 11, 13, 14, 15	1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20
Studying textbooks	N/A	1, 4, 5, 6, 7, 10, 12, 14, 17, 18, 19
Setting up teaching objectives	1, 2, 3, 4, 5, 6, 8, 11, 13, 14	1, 2, 3, 4, 7, 9, 10, 14, 18, 19, 20

difficulties in order to incorporate these factors into the design of lesson plans (see Table 3). CH20 elaborated:

> For new teachers, I generally ask them to first study students beforehand and then design their lesson plans based on students' reality. They should first study students' current conceptions such as their knowledge basis, prior experience, the intersection between the old and new knowledge points, and the interesting and difficult knowledge points for students. … As long as a new teacher obtains the habit of studying students, they can design lessons that are based on but higher than students' current thinking.

Ten U.S. teachers (62.5 %) also discussed that they would guide new teachers to attend to students, including their prior knowledge, student thinking, student difficulty, diversity, and inquiry (see Table 3). A closer inspection of the data, however, revealed an interesting cultural difference. Whereas Chinese teachers tended to study students before teaching, U.S. teachers mainly tried to understand students during or after teaching. This difference was more apparent in teachers' discussions of connecting new knowledge to students' prior knowledge. U.S. teachers referred to connections to prior knowledge as a result of rather than a condition for achieving instructional coherence. For example, US2 provided a typical response stating that coherence meant that a teacher understands that "in the end, the students connected the new ideas with what they already knew in a sensible manner." Such views on prior knowledge reflected the U.S. teachers' reasonable expectations for student learning outcomes, but they were quite distinct from the Chinese teachers' emphasis on studying students' prior knowledge beforehand in order to incorporate that information into planning a coherent lesson. Unfortunately, in this study, the Chinese teachers did not provide any information about how they would determine the students' prior knowledge beforehand.

Studying textbooks Eleven Chinese teachers (55 %) stressed the importance of intensive study of textbooks, which was the first thing that they would mentor new teachers to do (see Table 3). The purposes of studying textbooks included understanding the curriculum authors' intention about the essential knowledge points, and how they connected to prior and later pieces of knowledge. CH6's response was typical:

> A new teacher should first study textbooks identifying the location of the concept, that is, where this concept is from and what it leads to. Only after a thorough study of the lesson from the perspective of interconnected knowledge system can a teacher help students construct such a knowledge structure.

However, none of the 16 U.S. teachers mentioned the importance of studying textbooks in preparing and achieving coherent instruction (See Table 3). That the Chinese and U.S. teachers held different views of studying textbooks is not surprising (Cai, 2005; Cai & Wang, 2010), but the Chinese teachers' emphasis on studying textbooks did highlight a path that they viewed as key to achieving instructional coherence.

Setting up teaching objectives Ten U.S. teachers (63 %) and 11 Chinese teachers (55 %) emphasized that a coherent lesson should have a clear goal (see Table 3). US6 noted, "A coherent lesson would have a very clear direction stated in the beginning, as well as having authentic, challenging activities that directly align with the main goal or objective." Chinese teachers had a similar emphasis on teaching objectives. In addition, they uniformly used the popular term, "essential and difficult teaching points" (教学重难点), which was expected to "control the direction of coherence." (CH19).

4.2.2 Teaching stage

In addition to some overlap with the teachers' thoughts on the planning stage (e.g., understanding students), there were several new factors of instructional coherence that the teachers emphasized when discussing the teaching stage.

Managing a lesson structure Fourteen U.S. teachers (87.5 %) and 18 Chinese teachers (90 %) pointed out the importance of managing a lesson structure (see Table 4). The U.S. teachers tended to describe the activities to be done during each phase of the BME structure. US12's response is typical:

> If someone were to describe a lesson as coherent, I would think that the lesson had a logical "flow" to it, meaning that the lesson had a clear beginning, middle and an end. Often times, teachers begin a lesson with a warm-up, which could be a review of the previous night's assignment, a review of …. Then the lesson moves into the middle portion, the "meat" of the lesson. This portion is where students are learning new material. The middle portion may include some lecture or some teacher-centered time, but for the most part, it should be student centered… Finally, after the middle of the lesson, a coherent lesson has a closure. …. Closure could mean having students write down what they learned in the class, or what they had questions on.

Half of the U.S. teacher samples described the BME of a coherent lesson in a similar manner. For these teachers, it seemed that as long as a lesson had a teaching objective along with a complete BME structure or connected teaching phases, coherence could be achieved.

Indeed, when describing their BME structure, especially the middle part, many U.S. teachers emphasized "student-centered" activities, "controlled chaos," "moving students around" and "working in small groups." Some U.S. teachers (US2, US10, and US12) emphasized students' physical engagement (see Table 4). No Chinese teachers explicitly emphasized students' physical engagement as a means to achieve coherence for a lesson.

Although Chinese teachers did discuss the BME structure in the teaching phase, they seemed to possess a clearer image of and higher demands for a coherent lesson which went beyond the BME structure (except CH12). For example, the Chinese teachers emphasized careful design of teaching sequences, teaching language, and teacher questions beforehand. We elaborate each of these unique aspects below.

Table 4 Teachers' different emphases during the teaching stage

	U.S. teachers	Chinese teachers
Managing lesson structures	1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15,	1, 3, 4, 5, 6, 7, 8, 9, 10. 11, 12, 14, 15, 16, 17, 18, 19, 20
Attending physical engagement	2, 10, 12	N/A
Designing teaching sequence	N/A	3, 5, 9, 11, 12 14, 15, 19
Designing teaching language	10	5, 6, 7, 15, 16, 20
Designing questions	15	1, 5, 9, 10, 14, 19, 20
Challenging student thinking	3, 6, 10	1, 3, 7, 10, 11, 14, 17, 18, 19
Dealing with emerging events	13	4, 7, 9, 10, 13, 14, 17, 18, 20

Designing teaching sequences, language, and questions Eight Chinese teachers (40 %) stressed that the teaching content should be designed and arranged progressively (see Table 4). The frequently-used descriptors of ideal teaching included “rigorous organization of teaching with various levels” (CH5, CH14, CH15, and CH19), “appropriate levels of tightness” (CH3), and “combining the static and active, and the tight and loose” (CH9) (动静结合,张弛有度). CH9 suggested arranging tasks from easy to hard: “Such a design enables students to experience the process from being confused to clear. This aligns with students’ knowledge development and is beneficial for students’ construction of knowledge structures and developing their ability.”

Six Chinese teachers (30 %) emphasized the teacher’s use of language as an important part of instructional coherence (see Table 4). Three of them (CH5, CH6, and CH20) emphasized the teacher’s use of transitional language to make a nice transition from one activity to another. For instance, CH6 used a metaphor of a body joint, portraying two adjacent activities as *disjointed* if the wrap of the teacher’s transitional language was lacking. CH5 went even further, arguing that:

> A teacher’s coherent language is part of the instructional coherence. Especially in a transition between two activities, the use of nice transitional language could make the connection natural and smooth, which can facilitate students to understand the content in a coherent way.

These results were consistent with prior research findings (e.g., Wang & Murphy, 2004). In contrast to the Chinese teachers, none of the U.S. teachers mentioned the use of transitional language, although one teacher (US10) mentioned language accuracy.

In addition to language use, seven Chinese teachers (35 %) stressed the careful design of teachers’ questions to achieve coherence. Three teachers (CH1, CH9, and CH20) argued that teacher questions should be logical and consistent with students’ cognitive development. According to CH19, “Under the coherent and smooth flow, the teacher should set up a series of challenging questions to intrigue student enthusiasm of exploring the mathematical questions.” Although U.S. teachers expressed the same desire of instructional coherence, only one teacher (US15) mentioned that he would suggest that new teachers focus on their questions.

Challenging student thinking Nine Chinese teachers (45 %) and three U.S. teachers (20 %) expressed that instructional coherence should be achieved through challenging student mathematical thinking (see Table 4). For the Chinese teachers, the idea of challenging student thinking revealed an important distinction they were drawing between two kinds of coherence: surface coherence and real coherence. Surface coherence is indicated by a smooth flow enacting the planned lesson without challenging student thinking, whereas real coherence is marked by a smooth flow that challenges student thinking. CH19 explicitly contrasted these two kinds of coherence:

> If everything is unfolded according to exactly what the teacher designed, this is the coherence with surface smoothness… It (this kind of coherence) lacks beauty of up-and-down in student thinking and is not really helpful for student learning. Learning needs challenges. The teacher should design challenging questions one after another to achieve coherence through stimulating student thinking.

CH3 further pointed out that surface coherence that does not challenge student thinking can even be negative for learning because it “deprives students’ rights of their independent thinking and opportunities of raising their own questions.” Among the three U.S. teachers (US3, US6, and US10) who mentioned the importance of challenging student thinking, none of them explicitly differentiated between surface and real coherence.

Dealing with emerging events Nine Chinese (45 %) teachers and one U.S. teacher (6 %) argued that in order to achieve coherence, a teacher should deal well with emerging classroom events (e.g., unexpected student responses, questions, and difficulties, see Table 4). The Chinese teachers argued that how a teacher treats emerging events will yield different kinds of coherence. Given that emerging events are sometimes unpredictable, most Chinese teachers saw them both as a threat and an opportunity to achieve instructional coherence: "They [emerging events] may disrupt the teacher's predesigned coherent lesson… If the teacher can nicely integrate the unexpectedly emerging events into a planned lesson, this can make the instructional coherence really helpful for student learning" (CH4). Among the sixteen U.S. teachers, only US13 explicitly discussed the importance of dealing with emerging events and viewed emerging events as a great opportunity for learning: "There will always be situations that arise that are not planned for—that's the beauty of teaching kids and getting them involved. They should be thinking."

4.3 Conditions for a coherent lesson to foster student learning

When explicitly asked whether they agreed with the statement that a coherent lesson fosters student learning, the U.S. and Chinese teachers demonstrated different views, providing additional insight into how teachers view instructional coherence (see Table 5). The majority of U.S. teachers (87.5 %) expressed unreserved agreement (14 agreed, 1 partially agreed, 1 not sure). In contrast, the majority of the Chinese teachers took more conservative positions (5 agreed, 2 disagreed, and 13 partially agreed). They argued for specific conditions under which a coherent lesson might foster student thinking and learning. CH3's response was typical among the 13 teachers (65 %) who held partial agreement with the statement. She first described surface coherence and explained why it was not helpful for student thinking:

> If "coherence" refers to the effect that new learning causes students' smooth but surface thinking, such coherence indeed hinders students' thinking and deprives students' rights and the space to conduct independent thinking, especially the opportunity for posing questions. … The "non-smoothness and being challenged" are the necessary costs that students should pay for growth (CH3).

Further, CH3 described real coherence in her mind:

> On the other hand, if "coherence" refers to the following—the design of every lesson activity and event was closely related to students' questions, the lesson engages students into the process of co-constructing knowledge, the design has appropriate levels of tightness, and every student was completely engaged in thinking—such coherence will have positive effects on students' learning.

Table 5 Views on instructional coherence fostering student learning

		U.S.	CH
A coherent lesson fosters student learning.	Agree	1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12 ,13, 14, 15	2, 5, 12, 15, 16
	Partially agree	6	1, 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 19, 20
	Disagree	NA	17, 18
	Not sure	16	NA

CH17 was one of two teachers who disagreed with a definition of coherence as smooth teaching. This teacher viewed students' "emerging question, a curious eye-contact, or an unexpected student response" as valuable classroom resources, although they could make a pre-designed plan unsmooth. In fact, many other Chinese teachers also frequently emphasized "emerging events" and "emerging resources" (课堂生成), as noted above. Chinese teachers' conservative responses to the statement "coherence fosters learning" reflected that this group of Chinese teachers advocated real coherence with a strong awareness of the possible conflict between addressing students' thinking and the pre-designed plan.

5 Discussion

Although there are differences in the U.S. and Chinese teachers' views of instructional coherence, both groups of teachers in our study undoubtedly recognize that instructional coherence is important in effective teaching. As the purpose of this comparative study is to understand how teachers view the meaning of instructional coherence and the ways to achieve instructional coherence, our discussion focuses on these two aspects.

5.1 What does instructional coherence entail? Surface versus real

Both the Chinese and U.S. samples emphasized complete beginning, middle, and end (BME) structures, smooth teaching flow, and connected activities and topics for coherent lessons. The connected activities and complete lesson structure may ease students' comprehension and help them construct a textbase useful for remembering what was discussed during classroom teaching (Kintsch, 1986; Mannes & Kintsch, 1987; van Dijk & Kintsch, 1983). However, at both the micro and macro levels, instead of viewing instructional coherence as the connections of activities and smooth teaching flow (surface coherence), some Chinese teachers emphasized the underlying structures and knowledge connections embodied by these activities. Such emphases clearly align with the meaning of coherence in curriculum research (Schmidt et al., 2005) and with assertions in current research of the importance of attending to the "conceptual structure of mathematics" to improve teaching and learning (Richland, Stigler, & Holyoak, 2012). In addition, Chinese teachers stressed focusing on challenging students' thinking. Chinese teachers referred to the interconnected mathematical concepts and students' coherent and gradually deepened thinking as real coherence.

Although our findings about the Chinese teachers' emphasis on connected mathematical concepts and coherent student thinking are not new (e.g., Cai & Wang, 2010; Ma, 1999), the Chinese teachers' differentiation between surface and real coherence in the area of instructional coherence is innovative, insightful, and potentially makes a unique contribution to the field. This is because teachers who believe in real coherence value deeper connections among mathematical activities and topics. In this study, Chinese teachers who believed in real coherence expressed their willingness to detour from a smooth teaching flow to respond to emerging teaching events, such as unexpected student questions, and to raise questions to challenge students' thinking. Such a detour may encourage students actively to process the classroom discourse and make inferences among knowledge pieces, resulting in students' situation models that are the key to comprehension, learning and problem solving (Kintsch, 1986; van Dijk & Kintsch, 1983). Our findings also challenge the current field to rethink the essence of instructional coherence and, consequently, effective ways to achieve real instructional coherence.

5.2 How can teachers achieve instructional coherence? Pre-design versus emerging

Designing a lesson based on existing textbooks and student needs reflects pedagogical design capability, which refers to a "teacher's capacity to perceive and mobilize existing resources in order to craft instructional episodes" (Brown, 2009, p. 29). The intensive study of textbooks, in addition to being a first step in designing a coherent lesson, may also help teachers obtain a connected content knowledge base to reach instructional coherence. The results suggest that studying textbooks and studying student thinking beforehand are two of the most-mentioned approaches, albeit by Chinese teachers and not U.S. teachers, to achieve coherence in the design stage of lessons. The lack of pre-design and study of textbooks in U.S. teachers' lesson planning is a cause for concern if one hopes to improve teachers' ability to achieve instructional coherence (Cai, 2005). These findings challenge textbook designers to support teachers' lesson planning, teaching, and learning as they strive for instructional coherence.

Why do teachers (especially Chinese teachers) bother to design a lesson plan beforehand, given the possible tension between scripted plans and emerging events? According to discourse theory (van Dijk & Kintsch, 1983), any effective discourse should start with "design" even though unpredictable factors exist. The emerging events may disrupt surface coherence but still be an inseparable part of real coherence. This situational and dynamic view of coherence is especially true when we discuss instructional coherence, because in classroom teaching teachers discuss the emerging questions, capture students' current conceptions, and help students construct situation models to make inferences between current and prior knowledge (Kintsch, 1986; Mannes & Kintsch, 1987). As such, teachers pursue real coherence by emphasizing both the design of a lesson and the need to deal with emerging events.

5.3 Implications, limitations and future directions

The findings of this study contribute to our understanding of how teachers view instructional coherence and ways to attain it. However, our findings are based on a survey of a relatively small group of exemplary teachers, and thus should not be over-generalized. Nevertheless, our findings about the differentiation between surface coherence and real coherence shed some light on the essence of instructional coherence. In particular, this distinction suggests the importance of analyzing instructional coherence from a dynamic, co-constructive, and situational perspective.

For this study, we did not collect data on the teachers' classroom instruction because the purpose of the study was not to see how teachers practice instructional coherence, but to understand their ideas and conceptions about it. Thus, our findings rest only on an analysis of the teachers' stated views, not their actual practice. As such, this study contributes to future studies by providing some important comparative angles. For example, in addition to comparing instructional coherence between cultures, we should examine the consistency between teachers' views about instructional coherence and their actual teaching within cultures. Both consistency and inconsistency between teachers' views and their classroom instruction are particularly important in understanding relationships between mathematical teaching and cultural contexts (Cai & Wang, 2010; Skott, 2009; Speer, 2005). In addition, future studies could analyze classroom instruction to identify prototypes of lessons with surface and real coherence.

The findings from this study also have implications for classroom instruction and teacher professional development. For example, the study of textbooks appears to offer a starting point to improve teachers' pedagogical design abilities and their coherent knowledge base in order to reach real instructional coherence. It may also be helpful to focus on cultivating teachers' beliefs in the significance of lesson planning. Even though there will always be a tension between pre-designed lesson plans and emerging classroom events, working on both components may be a necessary path to improve pedagogical design capability and teaching flexibility. This in turn may help to manage the tension. Future studies are needed to explore strategies to help teachers improve their pedagogical design capability and flexibility to handle emerging events in the classroom.

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