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Creativity and mathematical problem posing: an analysis of high school students' mathematical problem posing in China and the USA

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Abstract In the literature, problem-posing abilities are reported to be an important aspect/indicator of creativity in mathematics. The importance of problem-posing activities in mathematics is emphasized in educational documents in many countries, including the USA and China. This study was aimed at exploring high school students' creativity in mathematics by analyzing their problem-posing abilities in geometric scenarios. The participants in this study were from one location in the USA and two locations in China. All participants were enrolled in advanced mathematical courses in the local high school. Differences in the problems posed by the three groups are discussed in terms of quality (novelty/elaboration) as well as quantity (fluency). The analysis of the data indicated that even mathematically advanced high school students had trouble posing good quality and/or novel mathematical problems. We discuss our findings in terms of the culture and curricula of the respective school systems and suggest implications for future directions in problem-posing research within mathematics education.

Keywords Advanced high school students · Cross-cultural thinking · Creativity · Geometry · Mathematical creativity · Novelty · Problem posing · Problem solving · US and Chinese students · Rural and urban Chinese students

1 Introduction

Creativity is a buzz word in the twenty-first century often invoked by policy makers, scientists, industry, funding bodies, and last but not least systems of education worldwide. In fact, the vision and/or mission statements of most school districts in the USA and Canada include the word “creativity” in it. Until recently, the last decade of published research

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includes only a handful of articles focused specifically on mathematical creativity (Leikin, Berman, & Koichu, 2010). This is even more amplified within the domain of mathematics education research in the scarcity of articles that tackle giftedness and/or creativity. For instance, in *Educational Studies in Mathematics* (*ESM*), one of the oldest journals in mathematics education, there are six articles that report on studies related to giftedness (high ability) and creativity in the last 40 years starting with Presmeg (1986). In 2010, two papers focused on creativity were published in ESM. Shriki (2010) tried to move beyond creativity as a process versus product dichotomy in a study involving 17 prospective mathematics teachers participating in a series of creativity awareness-developing activities. This study relied on teacher reflections as a way to understand how creativity awareness can be fostered among teachers. Bolden, Harries, and Newton (2010) used questionnaires and semi-structured interviews with preservice teachers in the UK, to resolve differences between "teaching creatively" versus "teaching for creativity," the latter of which required a deeper understanding of mathematical conceptual knowledge. Both these papers targeted prospective mathematics teachers. Other than the studies reported by Sriraman (2003, 2004, 2005, 2008, 2009) and Sriraman and Lee (2011), there are very few attempts to understand the nature of mathematical creativity in high school students when confronted with novel mathematical tasks. The present article continues this sequence of studies but from a cross-cultural viewpoint involving high school students in China and the USA.

2 Creativity research

2.1 A Terse survey

Creativity research in general is somewhat divisive and polar in its orientation. For instance, in psychology, some view it as effects of divergent thinking, while others view it as convergent thinking. Creativity is also viewed as domain specific by some and domain general by others (Plucker & Zabelina, 2009). The research literature on mathematical creativity has historically been sparse with an overreliance on the writings of eminent mathematicians of the nineteenth and twentieth centuries (Brinkmann & Sriraman, 2009; Sriraman, 2005). Mathematicians like Henri Poincaré (1948), Jacques Hadamard (1945), and Garrett Birkhoff (1956) have attempted to demystify the mathematician's craft and explain the mystery of "mathematical" creation (Sriraman, 2005). Early accounts of mathematical creativity (Hadamard, 1945; Poincaré, 1948) influenced by Gestalt psychology describe the creative process as that of *preparation–incubation–illumination and verification* (Wallas, 1926). A large part of the creative process remains a grey area so to speak, particularly the role of the unconscious in the incubatory period before any insight (or the Aha! moment) occurs. Paradoxically, these gestalt narratives *do not explain* the Gestalt or the whole of the creative process in any field per se and are also vague because they offer no insight specifically into the mathematician's mind. We have ample accounting and understanding of the starting and ending phases of creativity, but the "middle" phases, namely, incubation and illumination are still a topic of interest to psychologists, neuroscientists, and educators. Other reformulations of the incubatory phase are "endocept" which is defined as nonverbalized effects of (repressed) emotional experience (Ariete, 1976). Csikszentmihalyi (1996) coined the notion of "flow" to describe a middle phase of the creative process which is generative, that is, ideas are generated freely and affective dispositions described as fun, pleasure, even enrapture are found in the literature (Ghiselin, 1952). Psychiatric studies that have investigated the relationship between (highly) creative deviance and bipolarity describe flow as a type of mania (Andreasen & Glick, 1988; Richards, Kinney, Lunde, Benet, & Merzel, 1988).

More recently, a number of studies have specifically examined the role of an incubation period in creative problem solving. Sio and Ormerod (2007) conducted a meta-analytic[1] review of empirical studies that investigated incubation effects on problem solving and found that incubation is crucial in fostering insightful thinking. Psychologists term this *the fatigue hypothesis*, that is, the mind after a period of frenzied and intense activity requires a period of rest to overcome fatigue, and the relaxation during the period of rest results in new insights. According to this report and others similar to it (Vul & Pashler, 2007), understanding the role of the incubatory period may allow us to make use of it more efficiently in task designs to foster creativity in problem solving, classroom learning, and working environments. Mathematics educators try to incorporate incubation periods in classroom activity in temporal pauses during classroom discourse (Barnes, 2000) or extended time periods for problem-based learning (Sriraman, 2003). Incubation results in the positive effects of promoting students' creativity (Sriraman, 2004, 2005) and this seems to be self evident for mathematicians (Kaufman & Sternberg, 2006). There are recommendations based on this line of research that students should be encouraged to engage in challenging problems and experience this aspect of problem solving (Sriraman, 2008, 2009; Sriraman & Lee, 2011; Stillman et al., 2009).

2.2 Cross-cultural studies

During the past four decades, a large number of international evaluation studies of school mathematics have been conducted. In most of these studies, US students were outperformed by students in many other countries, especially students in East Asian countries. In most cross-national studies involving Chinese and US students' mathematics performance that have been reported (e.g., Husen, 1967; Robitaille & Garden, 1989), Chinese students outperformed their US counterparts. However, mathematics classes in China are often described as not conducive to effective learning (Wong, 2004). For example, the teaching method in the classroom was often described as “passive transmission” and “rote drilling” (e.g., Biggs, 1991). In order to understand this “paradox of the Chinese learner” (Huang & Leung, 2004, p. 348), many comparative studies have been conducted involving US and Chinese students (e.g., Cai, 1995, 1997, 1998; Ma, 1999; Stevenson, 1993; Stevenson & Stigler, 1992; Vital, Lummis, & Stevenson, 1988). But at the same time, it is widely accepted in China that US students are more creative in mathematics than Chinese students (e.g., National Center for Education Development, 2000; Yang, 2007). There are studies showing that US students are better than Chinese students in solving open-ended problems (e.g., Cai & Hwang, 2002) and in posing problems in mathematics (e.g., Cai, 1997, 1998). Therefore, more and more researchers have started looking at the strengths of US students' mathematics learning other than merely focusing on computational skills and routine problem solving. In general, there is a lack of literature addressing the differences in mathematical creativity between Chinese and US students or any other large-scale cross-national studies.

It is difficult to compare creativity in general terms between these two general populations due to significant cultural differences and difficulties of sampling comparable sets of students—the USA being perceived as a highly individualistic society where creativity is more or less a cultural norm, whereas China is perceived as a collectivist society where conformity is the norm (Hofstede, 1980). There are some large-scale empirical studies that examine temperamental differences between US and Chinese children ranging between the

[1] There were 117 studies included in this meta-analysis that most of them support the existence of incubation effects on problem solving.

ages of 9 and 15 that may shed light on cultural norms (Oakland & Lu, 2006). In Oakland and Lu's (2006) study analyzing[2] the temperamental dispositions on a bipolar spectrum (extroversion–introversion, thinking–feeling, practical–imaginative, and organized–practical) of 3,539 US students with 400 Chinese students of the same ages, the reported finding was that Chinese children preferred extroversion to introversion, practical to imaginative, thinking to feeling, and organized to flexible styles. They found that although Chinese and US children did not differ on extroversion–introversion styles, they differed on the three other temperamental styles with Chinese children more likely to prefer practical, thinking, and organized styles, which may very well be reflective of values prominent in either a collectivist or individualist society.

2.3 Creativity and problem posing

In Usiskin's (2000) eight-tiered hierarchy of mathematical talent, students who are gifted[3] and/or creative in mathematics have the potential of moving up into the professional realm with appropriate affective and instructional scaffolding as they progress beyond the K–12 schooling into the university setting (Sriraman, 2005). Therefore, gifted and/or creative students in mathematics have been of special interest to many researchers in the field of mathematics education. Hadamard (1945) posited the ability to pose key research questions as an indicator of exceptional talent in the domain of mathematics. This is consistent with the paradigm in psychology that creative thinking often manifests itself in divergent thinking abilities, and we develop our study within the well-defined framework of problem posing/finding or problem generating being a feature of divergent thinking and hence of creativity (Runco, 1994; Torrance, 1988). To this end, we review some of the related literature on problem posing found in mathematics education.

Krutetskii (1976) and Ellerton (1986) contrasted the problem posing of subjects with different ability levels in mathematics. In Krutetskii's study of mathematical "giftedness," he used a problem-posing task in which there was an unstated question (e.g., "A pupil bought $2x$ notebooks in one store, and in another bought 1.5 times as many."), for which the student was required to pose and then answer a question on the basis of the given information. Krutetskii argued that there was a problem that "naturally followed" from the given information, and he found that high-ability students were able to "see" this problem and pose it directly, whereas students of lesser ability either required hints or were unable to pose the question. In Ellerton's (1986) study, students were asked to pose a mathematics problem that would be difficult for a friend to solve. She found that the "more able" students posed problems of greater computational difficulty (i.e., more complex numbers and requiring more operations for solution) than did their "less able" peers.

According to Jay and Perkins (1997), "the act of finding and formulating a problem is a key aspect of creative thinking and creative performance in many fields, an act that is distinct from and perhaps more important than problem solving" (p. 257). Silver (1997) claimed that inquiry-oriented mathematics instruction which includes problem-solving and problem-

[2] Cross-national studies of temperamental styles are typically based on the Myers and Briggs theory of temperament and the associated psychometric test called Myers–Briggs Type Indicator (MBTI).

Oakland, Glutting, and Horton (1996) adapted the MBTI to detect cross-national differences in children aged 8 to17 years old on four bipolar temperament style dimensions, namely extroversion–introversion, practical–imaginative (MBTI's judging–perceiving), thinking–feeling, and organized–flexible (MBTI's judging–perceiving). The adapted test is called the Student Styles Questionnaire (see Oakland et al., 1996).

[3] We do not enter into a discussion of the definition of mathematical giftedness in this paper. This is a well-defined term in the research literature in gifted education. In this paper, the participants by virtue of their enrollment in the advanced mathematical courses were among the high achievers in their respective schools and included students of varying mathematical abilities.

posing tasks and activities can assist students to develop more creative approaches to mathematics. It is claimed that through the use of such tasks and activities, teachers can increase their students' capacity with respect to the core dimensions of creativity, namely, fluency, flexibility, and originality (e.g., Presmeg, 1986; Torrance, 1988)

The purpose of this study was to investigate mathematically advanced high school students' abilities in posing mathematical problems. Participants were junior or senior students (16–18-year-olds) in high school. As stated before, very few studies have specifically focused on high school students as opposed to preservice teachers. By focusing on these age levels, we aim to reveal the students' problem-posing abilities at their end of K–12 school education and, therefore, shed light on the students' creativity in mathematics after their K–12 school education.

This study reports part of a dissertation study from which the data for this paper were drawn (Yuan, 2009). Among the three tasks in the problem-posing test, only one is discussed and reported in detail in this paper. The study is also different from previous studies in the sense that we focus on problem posing as an important but overlooked and least understood aspect of mathematical creativity. In the history of mathematics, there are numerous papers considered as seminal not because they have proved a long-standing theorem, but because they opened up entirely new areas of mathematical inquiry such as Hewitt's (1948) paper on rings of continuous functions, in addition to Hilbert's (1900) famous 23 problems that shaped the twentieth century of mathematics.

2.4 Operationalizing problem posing as creativity

The topic of problem posing has been of interest to the research community in the past decades; however, there is a lack of theory concerning problem posing. In 1982, Dillon claimed that no theory of problem finding had been constructed and that there are several different terms such as problem sensing, problem formulating, creative problem discovering, and problematizing (Allender, 1969; Bunge, 1967; Taylor, 1972). Similarly, Stoyanova and Ellerton (1996) proposed that research into the potential of problem posing as an important strategy for the development of students' understanding of mathematics had been hindered by the absence of a framework which links problem solving, problem posing, and mathematics curricula. Building on Guilford's (1950) structure of the intellect, the framework proposed by Stoyanova and Ellerton classified a problem-posing situation as free, semi-structured, or structured. According to this framework, a problem-posing situation is referred to as free when students are asked to generate a problem from a given, contrived, or naturalistic situation (see example 1 below). A problem-posing situation is referred to as semi-structured when students are given an open situation and are invited to explore the structure of that situation and to complete it by applying knowledge, skills, concepts, and relationships from their previous mathematical experiences (see example 2 below). A problem-posing situation is referred to as structured when problem-posing activities are based on a specific problem (see example 3 below). All three examples below are taken from Stoyanova (1998). In this study, we made use of problem-posing activities to study mathematical creativity in advanced high school mathematics students, and compared to existing studies that report on either students identified as gifted, or prospective mathematics teachers; our focus is on groups of students with variations in high mathematical ability.

Example 1 Make up some problems which relate to the right angled triangle. (p. 64)

Example 2 Last night there was a party and the host's doorbell rang 10 times. The first time the doorbell rang only one guest arrived. Each time the doorbell rang after that, three more guests arrived than had arrived on the previous ring. Ask as many questions as you can. Try to put them in a suitable order. (p. 66)

Example 3 Some integers are arranged in the way shown below:

				1				
			2	3	4			
		5	6	7	8	9		
	10	11	12	13	14	15	16	
17	…	…	…	…	…	…	…	25

1. What would be the third number from the left of the 89th row of the accompanying triangular number pattern?
2. State other meaningful questions. (p. 70)

3 Methodology

3.1 Participants

According to Peverly (2005), even within the one country, different locations in China can vary greatly in terms of culture. Thus, this study selected students from two locations in China: Shanghai—an economically well-developed city in the south of China—and Jiaozhou—a small city in northern China that is considered as having strong historical roots in Confucian culture. Students from the USA were from Normal,[4] Illinois—a midwestern town in the USA. The US students in this study were from two advanced placement Calculus classes and two Precalculus classes. Those students were in the 11th or 12th grade.

In China, high school students usually are divided into two strands, namely, a science strand and an art strand. After the first semester in high school, students choose a strand and are assigned to different classes. Science strand students take more advanced mathematics courses in high school than arts strand students. In the school in this study in Jiaozhou, in each grade, there are two art strand classes and 10 science strand classes, two of which are "express" or accelerated science strand classes. Students in these two express science strand classes were admitted according to their achievement (total score of five subjects, namely, mathematics, Chinese literature, English, physics, and chemistry) in the high school entrance examination of the city, which they took after the ninth grade immediately before they entered the high school. The class in this study is one of the two 12th grade express science strand classes. Similarly to the Jiaozhou participants, the Shanghai participants in this study came from two 11th grade science strand classes that were the top two among the ten 11th grade classes in the high school. Therefore, the Chinese participants can be considered as advanced in mathematics.

Although participants in this study were from three very different locations, by choosing students from advanced classes in high schools in each of the three locations, the researchers managed to focus on mathematically advanced high school students in each of the three locations. Again, given the sampling difficulties in cross-cultural studies, we found the best fit given the constraints of the study. Initially, 68 Jiaozhou students, 73 Shanghai students, and 77 US students agreed to participate in this study. However, in the dissertation study from which the data of this paper were drawn, there were four tests to take and some students had to miss one or two of the tests; therefore, not all the participants' test papers were

[4] The reader may be surprised to learn that the term "normal" schools for teachers colleges comes from the first such school in Normal, Illinois.

analyzed. In the end, 55 Jiaozhou participants, 44 Shanghai participants, and 30 US participants were present for all the tests. Among the 30 US students, 17 were female and 13 were male; 17 were Advanced Placement Calculus Course students and 13 were Precalculus Course students. Among the 44 Shanghai students, 19 were female and 25 were male; all of the Shanghai students were in the 11th grade. Among the 55 Jiaozhou students, 18 were female and 37 were male; all of the Jiaozhou students were in the 12th grade.

3.2 Measures and instrumentation

The measures and instrumentation in this study include a mathematics content test and a mathematical problem-posing test. Both tests were translated into Chinese for the participants in China. Several pilot tests were conducted before they were used for the study.

3.2.1 The mathematics content test

The purpose of the mathematics content test in this study was to measure the participants' basic mathematical knowledge and skills. Instead of developing a test for this study, the researchers adapted the National Assessment of Educational Progress (NAEP) 12th grade Mathematics Assessment as the mathematics content test because this assessment fits the purpose of the study very well. NAEP is the only nationally representative and continuing assessment of what America's students know and can do in various subject areas (National Center for Educational Statistics, 2009). The 2005 mathematics framework focuses on two dimensions: mathematical content and cognitive demand. By considering these two dimensions for each item in the assessment, the framework ensures that NAEP assesses an appropriate balance of content along with a variety of ways of knowing and doing mathematics. The 2005 framework describes four mathematics content areas in high school: number properties and operations, geometry, data analysis and probability, and algebra.

3.2.2 The mathematical problem-posing test

Using Stoyanova and Ellerton's (1996) framework of mathematical problem posing, three situations were included in the mathematical problem-posing test, namely, free situation, semi-structured situation, and structured situation. The mathematical problem-posing test was developed based on Stoyanova's (1997) and Cai's (2000) research. There are three tasks in the mathematical problem-posing test:

Task 1 *Free problem-posing situation:* There are 10 girls and 10 boys standing in a line. Make up as many problems as you can that use the information in some way.

Task 2 *Semi-structured problem-posing situation:* In the picture below (Fig. 1), there is a triangle and its inscribed circle. Make up as many problems as you can that are in some way related to this picture. The problems could also be real-life problems. Again, do not limit yourself to the problems you have seen or heard of—try to think of as many possible and challenging mathematical problems as you can.

Task 3 *Structured problem-posing situation*: Last night there was a party at your cousin's house and the doorbell rang 10 times. The first time the doorbell rang only one guest arrived. Each time the doorbell rang, three more guests arrived than had arrived on the previous ring.

1. How many guests will enter on the 10th ring? Explain how you found your answer.
2. Ask as many questions as you can that are in some way related to this problem.

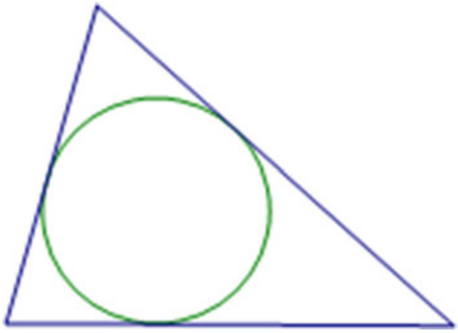

Fig. 1 Semi-structured problem posing situation

To encourage participants to try their best in posing mathematical problems, the following scenario was added in the beginning of the problem-posing test. Imagine that your school is participating in a problem-posing competition in mathematics among all the high schools in town. The schools that generate the most problems or/and the best quality problems will be rewarded. In addition, the students who pose the most number of problems or/and the best quality problems will be rewarded. Last week, student Jenny from another high school created five really good problems for each of the three situations below. Jenny also bragged that no one else could do better than she did. Now, try to prove her wrong by making up as many problems as you can. Do not limit yourself to the problems you have seen or heard of—try to think of as many possible and challenging mathematical problems as you can.

In the larger study from which the data of this paper were drawn (Yuan, 2009), participants' responses to the problem-posing test showed that the contexts of task 1 and task 3 had a significantly different influence on the participants' thinking processes due to the differences in the participants' culture (Van Harpen & Presmeg, 2011). For example, a Chinese student posed the following problem for task 1. It is a common practice for a class to have a monitor, who helps the teachers to keep the students well behaved, and a class representative, who helps the subject teacher to hand out and collect student work. That is not a common scenario in the USA.

> Problem: A class of 10 students are to select a monitor, a Chinese class representative, and a mathematics class representative. How many different ways of filling in the three positions are there? One person can at most take two jobs.

A US student Deanna posed a problem about parking cars for task 3. In China, different from in the USA, it is not common for people to have their own cars or trucks.

> Problem: If they parked their cars in a straight line, how long would it be? 1/2 of the guests drove 6 feet cars, 1/4 of them drove 5 feet cars, and 1/4 of them drove 9 feet trucks.

Since task 2 involves only a geometric figure, it directed participants' attention to be more focused on the mathematics than the other two tasks. As a result, task 2 is more culturally fair to the participants in the three groups. For this reason, analysis of data from task 2, the semi-structured problem-posing situation, is reported here.

3.3 Interview with the students

Eight students in the Jiaozhou group, 12 students in the Shanghai group, and 12 students in the US group were interviewed. The purpose of the interviews was to find how the problems were generated so that the researcher could see the differences in the mathematical problem-posing processes between the three groups. All interviews were conducted by the first author. The interviews were audio-taped and transcribed.

3.4 Data collection procedures

Since the researchers were based in the USA, both tests were administered in Chinese by the mathematics teachers of the classes in each of the two locations in China. With the US students, the principal researcher (the first author) conducted the mathematical problem-posing test in person. The mathematics content test was given by the mathematics teacher of the class due to a time conflict. The test does not require any instruction other than handing out of test papers, timing, and collecting test papers. The working time for the mathematical problem-posing test and the mathematics content test were both 50 min for all the students.

3.4.1 Data collection in China

In the Shanghai high school, students have a 50-min self-study period between lunch and the first class period in the afternoon. The two tests were conducted 2 weeks apart during the self-study period. In the Jiaozhou high school, students are required to attend four 50-min self-study periods every Saturday morning. The two tests were given 2 weeks apart during the Saturday morning self-study period. The mathematics content test requires that each student have the same set of tools, including a measuring ruler, a protractor, a spinner, etc. The tools were purchased in the USA and were sent to China before the tests were conducted. The test was sent to the teachers through email and the teachers then printed the tests ready to be used. Similarly to the mathematics content test, the mathematical problem-posing test was also sent to the teachers in China via email and the teachers then printed them ready to be used with the students. No tool was needed for this test.

3.4.2 Data collection in the USA

Because the US participants in this study were at a university school, where teachers and students were more willing to participate in educational research, the teacher allowed tests to be conducted in regular class time. The same tools described above were provided for the mathematical content test. The reason that this test can be given by a different person is that this test does not require any instructions other than handing out test papers, timing, and collecting test papers.

3.5 Data analysis procedures

3.5.1 Data analysis for the mathematics content test

There are 50 items in the mathematics content test. After ranking the individual scores, a Kruskal–Wallis test was used to evaluate differences among the three groups. Also, Mann–Whitney U test was used to evaluate differences between each pair of the three groups.

3.5.2 Data analysis for the mathematical problem-posing test

The problems posed by the participants in the mathematical problem-posing test were first judged as to their viability. Responses that are not viable were eliminated from further consideration. For example, responses such as “find the area of the circle” without any other additional information were eliminated. The remaining responses that are viable were scored according to the rubrics in terms of their fluency and flexibility. The rubrics were developed by the researchers as described below.

All the responses were typed into a Microsoft Word document and response frequencies were recorded. The responses generated by the three different groups of students were separated so that the researchers could see the differences among the groups.

Responses were categorized according to the type of questions posed in the problem. There are also problems that are difficult to fit in any of the categories. The researchers decided to put them in a category called "others." Table 1 gives an example of each category.

Categories formed by the principal researcher were checked by the co-researchers. Any differences between the researchers' coding were discussed and the categories were refined over the course of 3 months in the summer of 2008. The responses of students from the three groups to the mathematical problem-posing test were then placed into these three categories. However, it turned out that some categories were not used in all three groups. For example, Jiaozhou students posed dilation problems (e.g., construct a figure twice as big as the original one using a ruler and a compass), but US students and Shanghai students did not. Therefore, the rubric was refined by subsuming some categories into others. For example, the dilation category in the Jiaozhou rubric was subsumed into the transformation category in the common rubric. The total number of viable problems generated by a student is defined as his/her fluency score. The total number of categories that a student's viable problems involve is defined as his/her flexibility score and was not necessarily the same as the fluency score (see Table 1 for examples of the mathematical problem-posing test categories).

The originality of each of the responses was then determined according to their rareness. Since students in the three groups have different textbooks and instruction, one rare response in one group might not be rare in another group. Therefore, the originality of the responses was relative to other students in the same group. For that reason, the originality was analyzed separately among the three groups and was not compared across groups.

Table 1 Examples of the mathematical problem-posing test categories

Categories	Examples
1. Analytical geometry	In triangle *ABC*, the coordinates of the vertex are given, *B*(0,0), *A*(2,1), and *C*(5,−1). *BD* is the height. Find the equation of *BD*
2. Lengths	If the triangle is a right triangle, the hypotenuse is 2, another angle is 60°, find the radius of the circle
3. Area	Given the radius of the circle *r*, find out the minimum area of the triangle
4. Angles	Construct two perpendicular segments from the center of the circle to the two sides of the triangle. The angle formed by the two segments is 120°. Find the angles of the triangle
5. Transformation	What degree will the triangle have to rotate for point *A* to be where point *B* is?
6. Involving auxiliary figures	Draw a tangent line of the circle and intercept the triangle at *D* and *E*. The vertex of the triangle between *D* and *E* is *M*. Find the range of *MD*/*ME*
7. Three-dimensional	The radius is 5. What is the maximum volume of a ball that can go through?
8. Probability	If you are to drop something to the circle, what is the probability of it falling into the triangle?
9. Proofs	Given triangle *ABC*, and *D*, *E*, *F* are the midpoints of *AB*, *BC*, and *CA*. Prove that $AD=AF$ and $\|AB - AC\| = \|BE - EC\|$
10. Others	Plant 6 different flowers in the four areas and no adjacent two can be the same color. How many different ways?

The researchers decided that if one problem was posed by 10 % or more of the participants in the corresponding group, the problem would be considered as not original. In addition, there are problems that were posed by less than 10 % of the total number of participants but were not considered as original. For example, the following problem is not considered as original because the mathematics involved in the problem is at a very low level to a high school student.

> If there are four girls with brown hair and two more boys with brown hair than girls, how many people do not have brown hair?

In scoring the responses generated by the students in this study, two researchers scored the same six copies of test papers and compared the scores.

4 Results

4.1 Results of the mathematical content test

There are 50 items in the mathematics content test. The averages of the three groups are 45.8 for Jiaozhou group, 36.2 for Shanghai group, and 36.5 for US group. The outcome of the Kruskal–Wallis test indicated significant differences among the three groups, $H=82.131$ (2, $N=129$), $p<.05$ two-tailed. The outcome of the Mann–Whitney U test indicated that there are statistically significant differences between the US group and the Jiaozhou group (Mann–Whitney $U=64$, $n_1=30$, $n_2=55$, $p<.05$ two-tailed), that there are significant differences between the Shanghai group and the Jiaozhou group (Mann–Whitney $U=64$, $n_1=44$, $n_2=55$, $p<.05$ two-tailed), and that there is no significant difference between the US group and the Shanghai group (Mann–Whitney $U=653$, $n_1=30$, $n_2=44$, $p>.05$ two-tailed).

4.2 Results of the mathematical problem-posing test

Results of the mathematical problem-posing test are presented for analyses of fluency, flexibility, and originality.

4.2.1 Fluency

As discussed above, in analyzing the problems generated by the students, the problems that are non-appropriate (e.g., How old are the children?) and problems that lack the information needed to determine a solution (e.g., How many girls and how many boys are there at the party?) were excluded from further analysis. Those problems were considered as nonviable problems. Table 2 presents the number of viable problems posed by students from each of the three groups. Since the numbers of students in each of the three groups were different, the average percentage of nonviable problems generated by the students in each group was calculated with the following division:

$$\frac{\text{Number of nonviable problems} \times 100\%}{\text{Number of nonviable problems} + \text{number of viable problems}}.$$

It should be pointed out that the criteria classifying problems as viable or nonviable were based on the researchers' judgment. It was found that 31 % of the US students' problems,

Table 2 Comparison of students' fluency scores

	US students	Shanghai students	Jiaozhou students
Mean of fluency scores	4.6	2.0	4.9
Median of fluency scores	4	1.5	5

42 % of the Shanghai students' problems, and only 15 % of the Jiaozhou students' problems were nonviable problems. Many students posed problems such as "what is the area of the circle" or "what is the area of the triangle" without giving the measures of the radius or the sides. Notice that Jiaozhou students posed the least percentage of nonviable problems, which means that Jiaozhou students tended to give necessary information for the problems to be solvable.

After the nonviable problems were eliminated, all the viable problems were analyzed for their triviality. For example, the following problem is considered as a trivial problem since the mathematics involved in the problem is at a very low level for a high school student.

If the diameter of the circle is 32, what is the circumference?

The percentages of trivial problems were calculated as follows:

$$\frac{\text{Number of trivial problems} \times 100\%}{\text{Number of viable problems}}.$$

It was found that 9 % of the US students' viable problems, 8 % of the Shanghai students' viable problems, and 6 % of the Jiaozhou students' viable problems were trivial problems.

4.2.2 Flexibility

In counting the number of problems generated by the students in each group, the same problems generated by the same group of students were counted once (Table 3). For example, the following two problems were counted as one problem and were categorized as, "Given the three sides of the triangle, find the area of the inscribed circle."

Problem 1 Given that the three sides of the triangle are 3, 4, and 5, find the area of its inscribed circle.

Problem 2 Given that the three sides of the triangle are 5, 6, and 7, find the area of the circle.

Table 4 shows the distribution of the different categories posed by different groups of students. Consistently, for the three groups, the categories with the greatest number of responses are length and area.

However, not all the three groups posed problems for all 10 categories. The US students did not pose problem involving categories transformation and proofs. The Shanghai students did not pose problems involving categories analytical geometry, transformation, probability, and proof. The Jiaozhou students posed problems that covered all the 10 categories.

Table 3 Comparison of students' flexibility scores

	US students	Shanghai students	Jiaozhou students
Mean of flexibility scores	3.9	1.6	4.1
Median of flexibility scores	4	1	4

As to category analytical geometry, only 0.9 % of the US students' problems were in this category and none of the Shanghai students posed problems of this category. Jiaozhou students, different from the other two groups, posed 11 problems in the analytical geometry category. See the following problem for an analytical geometry example:

Points B and C are fixed. Point A is movable. $|BC| = 4$ and $|AC| - |AB| = 2$. Find the locus of A.

Another observation is that both Shanghai students and Jiaozhou students posed a relatively high percentage of problems that involve other figures (14.1 and 12 %), while a low percentage of problems were posed by the US students (2.8 %). For example,

1. *Adding lines:* Draw a tangent line of the circle and intercept the triangle at D and E. The vertex of the triangle between D and E is M. Find the range of MD/ME.
2. *Adding triangles:* If there is an inscribed triangle similar to the original one, find out the ratio of the area of the two triangles.
3. *Adding circles:* If the triangle is inscribed in another circle, find the ratio of the area of the two circles.
4. *Adding quadrilaterals:* AB, BC, and AC are given. Build a rectangle in the circle. Find the rectangle with the largest area.

In one further example of difference between groups, Jiaozhou students posed 10 problems in the category of proof.

Given triangles ABC, D, E, and F are the midpoints of AB, BC, and CA. Prove that $AD = AF$ and $|AB - AC| = |BE - EC|$.

Table 4 Distribution across categories of the three groups' viable problems

	US students	Shanghai students	Jiaozhou students
Analytical geometry	1 (0.9 %)	0	11 (5.5 %)
Lengths	39 (36.8 %)	27 (38 %)	48 (24 %)
Area	44 (42 %)	22 (31 %)	61 (30.5 %)
Angles	3 (2.8 %)	2 (2.8 %)	8 (4 %)
Transformation	0	0 (0 %)	1 (0.5 %)
Involving other figures	3 (2.8 %)	10 (14.1 %)	24 (12 %)
Three-dimensional	6 (5.7 %)	1 (1.4 %)	14 (12 %)
Probability	3 (2.8 %)	0	8 (4 %)
Proofs	0	0	10 (5 %)
Others	7 (6.7 %)	9 (12.7 %)	15 (7.5 %)
Total	106	71	200

Distribution of subcategories Some of the response categories are subdivided into subcategories. A closer look at those categories shows that within the subcategories, the distribution is very different, too. For example, seven subcategories appear within the lengths and area categories (as shown in Tables 5 and 6). Therefore, although lengths and area are the top two categories for all the three groups, the distribution of the subcategories varies greatly.

Table 5 shows that Jiaozhou students posed a lower percentage of problems of subcategories that involve finding the lengths of the sides of the triangle, the height and perimeter of the triangle, and the circumference of the circle. Those are more “straightforward” problems. Instead, Jiaozhou students seemed to focus more on subcategories that involve finding the radius or the circle, other quantities related to lengths, and problems that involve real-life contexts. Another finding is that Shanghai students did not pose problems that involve real-life contexts. That might indicate the preference in their mathematics instruction.

Table 6 shows that both Shanghai students and Jiaozhou students posed more than 25 % of their area problems in the subcategory involving ratio, while US students posed more than 30 % of their area problems in the subcategory involving the difference between the two areas. Again, Shanghai students did not pose problems involving real-life contexts.

4.2.3 Originality

As mentioned earlier, a problem was designated as not original if it was posed by 10 % or more of participants in that group. Results of this analysis have been presented elsewhere (Yuan & Presmeg, 2010; Yuan & Sriraman, 2011). Below are three examples that are considered as original problems according to the criteria within each group. For the US group, in which there are totally 30 participants, if one response was posed by three or more than three participants, which is more than but including 10 % of the 30 participants, then it is considered as not original. For the Shanghai group, in which there are totally 44 participants, the researchers decided that if one response was posed by four or more than four participants, which is about 10 % of the 44 students, then it is considered as not original. For the Jiaozhou group, in which there are totally 55 participants, the researchers decided that if one response was posed by six or more than six participants, which is about 10 % of the 55 students, then it is considered as not original. Below are three examples that are considered as original problems according to the criteria within the group. See Yuan and Presmeg (2010) and Yuan and Sriraman (2011) for more details on the originality of the posed problems.

Table 5 Distribution of subcategories of category *length*

	US students	Shanghai students	Jiaozhou students
Lengths of the sides or the perimeter of the triangle	11 (28.2 %)	7 (25.9 %)	2 (4.1 %)
Circumference of the circle	6 (15.4 %)	5 (18.5 %)	2 (4.1 %)
Ratio of the triangle's perimeter and the circle's circumference	1 (2.5 %)	0	4 (8.2 %)
Radius of the circle	5 (12.8 %)	6 (22.2 %)	16 (32.7 %)
Height of the triangle	3 (7.7 %)	3 (11.1 %)	0
Other quantities related to lengths	2 (5.1 %)	6 (22.2)	16 (32.7 %)
Involving real-life contexts	11 (28.2 %)	0	9 (18.4 %)
Total	39	27	49

Table 6 Distribution of subcategories of category *area*

	US students	Shanghai students	Jiaozhou students
Area of the triangle	9 (20.5 %)	11 (50.0 %)	8 (13.1 %)
Area of the circle	8 (18.2 %)	2 (9.1 %)	21 (34.4 %)
Area of the difference between the triangle and the circle	14 (31.8 %)	2 (9.1 %)	2 (3.3 %)
Sum of the areas of the triangle and the circle	1 (2.3 %)	1 (4.5 %)	1 (1.6 %)
Involving ratio of the areas	3 (6.8 %)	6 (27.3 %)	18 (29.5 %)
Other quantities related to areas	0	0	1 (1.6 %)
Involving real-life context	9 (20.5 %)	0	10 (16.4 %)
Total	44	22	61

A US example: If $AC=100$ m, $AB=30$ m, and $BC=75$ m, what is the circumference of the circle? What if the triangle was inscribed inside a circle?

A Shanghai example: If the perimeter of the triangle is 20, find the maximum and minimum value of the circumference of the circle.

A Jiaozhou example: Given the sum of the three sides of the triangle ABC m, the center of the circle O, find the range of $|OA|+|OB|+|OC|$.

Some participants who posed original (i.e., rare) problems were interviewed to find out the thinking process in their problem posing. For example, one question asks participants which problems they posed were creative and why they thought so. US student Kurt reported the following problem as creative.

What is the perimeter of the triangle if the diameter of the circle is 1?

Kurt explained that “(It's creative) just because a lot of theorems are involved to get to the right answer.”

Shanghai participant Zhenyu posed the following problem.

In the right triangle ABC, A (0, 3), B (4, 0). The circle is inscribed in the triangle. If point P starts moving from point B to A, when $|PC|+|PB|$ reached its maximum, what are the coordinates of point P?

Zhenyu likes the above problem and thinks it is creative because “it involves motion.”

Jiaozhou participant Yanan posed the following problem:

If the two sides of the triangle are 3 and 6, find out the perimeter of the triangle when the area of the inscribed circle is the maximum.

Yanan explained that “I think it is creative to involve the area of the circle and the perimeter of the triangle.”

5 Discussion and concluding points

In the problem-posing test, the students were told, “Do not limit yourself to the problems you have seen or heard of—try to think of as many possible and challenging mathematical problems as you can.” Despite that information, students from the three groups were not able to pose many challenging problems. The creativity of students' responses was analyzed according to their fluency, flexibility, and originality. Some of the problems posed by the students were not viable because they lacked necessary information to find a solution. Among the viable problems, some were trivial because they were not challenging. In other words, students' scores on fluency were not as high as expected. The analysis of flexibility showed that, although students posed problems of diversity as a group, most of the problems focused on two main categories, area and length. Although scores on originality were not compared across groups, interviews with students who posed rare problems revealed a variety of reasons why the problems were considered as creative.

The findings of this study suggest that, despite the emphasis placed on this topic by the educators and governors in the USA and China (e.g., National Council of Teachers of Mathematics, 1989, 2000; Mathematics Curriculum Development Group of Basic Education of Education Department, 2002), problem posing is not yet an established element in instruction in the classrooms. In addition, even though the participants in this study were from advanced mathematics courses in high school, they did not perform very well on the mathematical problem-posing test. This suggests that students who are good at solving routine mathematical problems or taking routine mathematical tests might not be good at posing mathematical problems. Below, the authors attempt to explain the findings from different perspectives and also suggest future directions in research on mathematical problem posing.

5.1 Influence of curricula

The differences in the three groups of students' performances on the problem-posing test can at least partly be explained by the differences in the mathematics content they have learned. As mentioned earlier, participants from Jiaozhou, China, were in 12th grade. These students have taken topics such as introductory 3-D geometry, introductory analytical geometry, 2-D vectors, and transformation in their first year of high school. Since these students are in the science strand, they have also taken 3-D vectors and 3-D geometry in their second or third year of high school. The curriculum structure in Shanghai is very similar to that in Jiaozhou. However, since the Shanghai participants were in the 11th grade, which is the second year in high school, they have not taken as high level geometry courses as Jiaozhou participants had. In the US high school, students take geometry in their first year where they are introduced to the basic postulates and theorems of geometry. Students who take precalculus also study plane and solid analytic geometry in their third year. Since the US students in this study were in precalculus (third year) or AP calculus course (fourth year), they should have studied high level geometry content over the years.

The results of the mathematics content test suggest that, although the participants were all taking advanced courses in their school, the US participants and Shanghai participants' basic mathematical content knowledge is not as strong as the researchers expected. That might be explained by the following differences. Jiaozhou students were in their last year of high school and they would need to take the college entrance examination in 6 months. Therefore, they needed to sustain their knowledge till the end of their high school. Shanghai students would take the college examination in 18 months and had not learned all the mathematics content yet. US students did not need to take any college entrance examination. These differences suggest that Jiaozhou students' mathematics was stronger when they were tested and that also might help explain why Jiaozhou students posed problems of more diversity than the other two groups, more problems of the “analytical geometry” category than the other two groups. The differences in the distribution of the categories of posed problems suggest that the problems posed by students might be related to students' background mathematical knowledge. In a sense, this echoes the claim that basic knowledge and basic skills in mathematics could be highly related to creativity in mathematics (Zhang, 2005), as opposed to viewing basic skills as rote or non-creative.

5.2 Implication of relationships between students' mathematical basic knowledge and mathematical problem-posing abilities

The findings from this study indicated that there are differences in the mathematical problem-posing abilities among the three groups. The Jiaozhou group posed fewer nonviable problems and fewer trivial problems than the Shanghai group and the US group. This result contradicts those found by Cai and Hwang (2002), who studied sixth graders' mathematical problem posing and found out that, although Chinese students did better in computation skills and solving routine problems, US students performed as well as or better than those Chinese students in problem-posing tasks. Again, the implication is that students' problem-posing abilities might be affected by their mathematical knowledge. Students from Jiaozhou in this study scored much more highly than the other two groups in the mathematics content test and the Jiaozhou students also did much better in the mathematical problem-posing test. The superior performances of Jiaozhou students in the mathematics content test and the mathematical problem-posing test suggest that there might be some relationship between the two.

In fact, in China, educators (e.g., Zhang, 2005) have reflected on the mathematics education in the past and claimed that the basic knowledge and basic skills in mathematics might or might not be highly related to creativity in mathematics, but there is a kind of balance between them. Wong (2004, 2006) summarized the characteristics of the Confucian Heritage Culture learners' phenomenon and pointed out that the Chinese students' focus on the basics might be related to the ancient Chinese tradition of learning from “entering” to “transcending the way.” Wong's observation echoes that of Gardner's (1983) that imitating the master is the starting point of the path to becoming the master one day. Future research in the relationships between mathematics content knowledge and mathematical problem posing will help to validate the observations by Wong and Gardner.

5.3 Limitations of this study

In this study, the participants were selected from three locations, a big city in China, Shanghai, a small city in China, Jiaozhou, and a town in the USA. Shanghai students were in the 11th grade. Jiaozhou students were in the 12th grade. Some of the US students were in the 11th grade and some were in the 12th grade. The students in the three locations do not have the same mathematics curriculum. Thus, the differences in the mathematical background and contexts of the three groups constituted a limitation of this research. In addition, the students were not selected randomly within the three student populations. Therefore, the findings of this study cannot be generalized to other students in the three locations.

Also, since the principal researcher of this study was based in the USA, she could not go to China to implement the tests in person. The tests given to the Chinese students in this study were all administered by the classroom teachers. Thus, it is hard to know how seriously the Chinese participants treated the tests. For example, Shanghai students took the tests during their self-study period between lunch and the first class period in the afternoon, and they also did poorly on both the mathematical problem-posing test and the mathematics content test. That indicates that some students might not have done their best on the tests due to fatigue or attitude.

5.4 Importance of problem-posing research

Problem-solving research has often been criticized as having reached an impasse (English & Sriraman, 2010). Polya's (1945) oft cited work provided the impetus for the ensuing research that took place in the following decades, which included a focus on novice versus expert problem solving (e.g., Anderson, Boyle, & Reiser, 1985), problem-solving strategies and meta-cognitive processes (e.g., Lester, Garofalo, & Kroll, 1989), and problem posing (English, 1997; Brown & Walter, 1983). However problem posing has not received the same attention as the other aforementioned areas. Problem posing has been researched to an extent with younger learners in the context of combinatorial situations (Sriraman & English, 2004), and more recently, problem posing has come to the foreground in the area of mathematical modeling in the elementary and middle grades (English, 2007), but in general has received scant attention as an aspect of mathematical creativity. This study indicates the necessity for more inquiry into this line of research within mathematics education, in which learners are presented with problem-posing opportunities in different areas of school mathematics, with the goal of stimulating creativity in intra-mathematical thinking as demonstrated by the Jiaozhou students, as well as diverse mathematical thinking to generate problems that are contextually different. A larger goal of bringing problem posing to the foreground in the study of mathematical creativity is to develop culturally congruent instruments that can be used to conduct larger empirical studies that compare cross-national differences. This study can be viewed as a starting point in this direction.

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