Procedural variation examples Fluency project Enigma Maths Hub

A collection of procedural variation questions that we have been using to develop mastery in mathematical fluency with our pupils.

Created by a group of schools involved in a fluency project funded by the Enigma Maths Hub

Background

In order to develop fluency with our pupils we explored the use of procedural variation.

Procedural variation is often used in Shanghai and we have been hearing more about this approach in the UK.

Mike Askew has talked about Variety or Variation and we used this when developing our own understanding



Variety or Variation?

Mike Askew uses the following statements which we found useful

Variety

'Pick and mix'
Most practice exercises contain variety

Variation

Careful choice of WHAT to vary

Careful choice what the variation will draw attention to



Addition and subtraction variation examples



Variation examples Fluency project Enigma Maths Hub Caroline Haslett Primary School Louise Cullen Foundation Stage



Foundation Stage and variation

This activity was carried out by FS children as a morning activity.

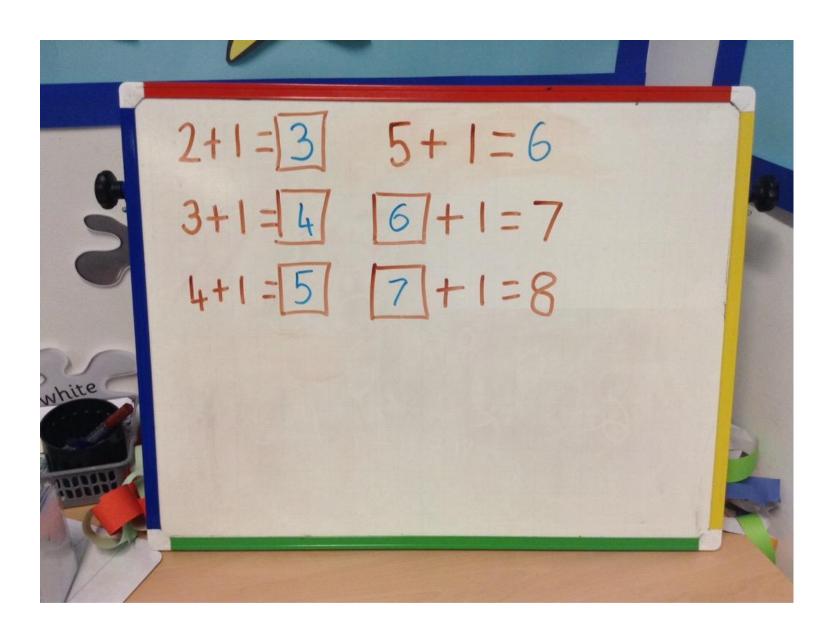
Some children were able to spot the pattern but found it difficult to verbalise what they did.

T said "there is a 6 there and a 6 there". S said "1, 2, 3, 4, 5, 6"

Although children were generally unable to verbalise what they did, I think several of them did use the pattern as they got the last two missing numbers correct which I would not have expected them to.



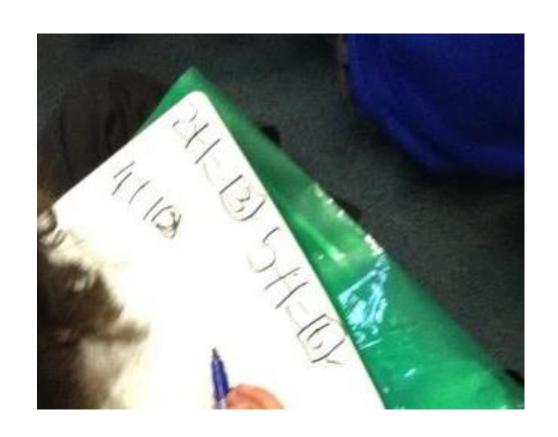
Foundation stage and variation





Foundation stage and variation

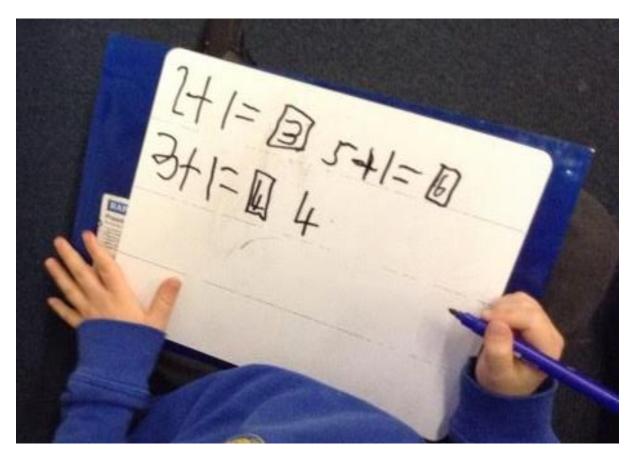






Foundation stage and variation







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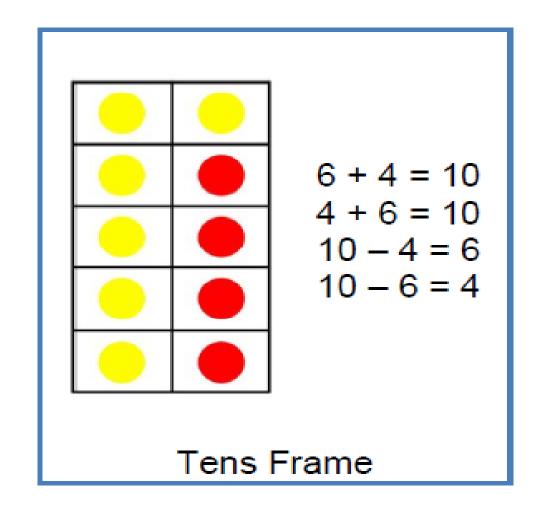


Year 1 Variation: Facts of 10

1 + 9 =	2 + 8 =	3 + 7 =
+ 1 = 10	+ 2 = 10	+ = 10
+ 9 = 10	+ 8 = 10	7 += 10
10 = 9	10 = 8	10 = 7
10 = 1	10 = 2	10 = 3
6 + 4 =	5 + 5 =	
. 6 40		
+ 6 = 10	+ = 10	
+ 6 = 10 + 4 = 10	+ = 10 5 + = 10	
+ 4 = 10	5 += 10	



Variation supported through model below



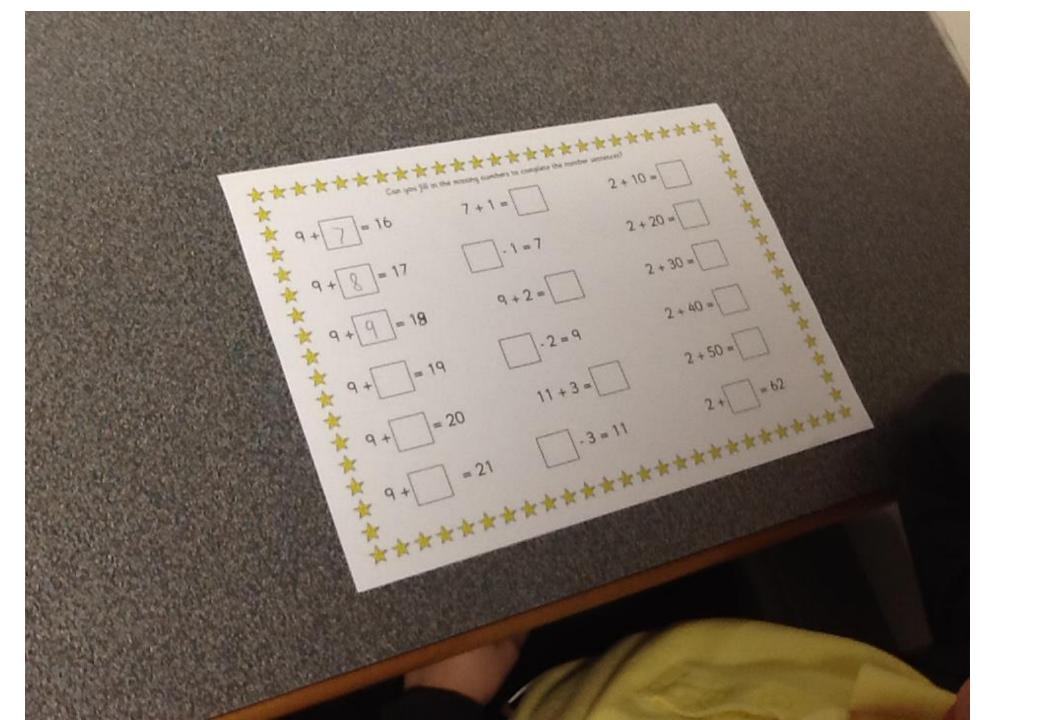


Variation examples Fluency project Enigma Maths Hub Barleyhurst Primary School Gemma Anderson Year 1

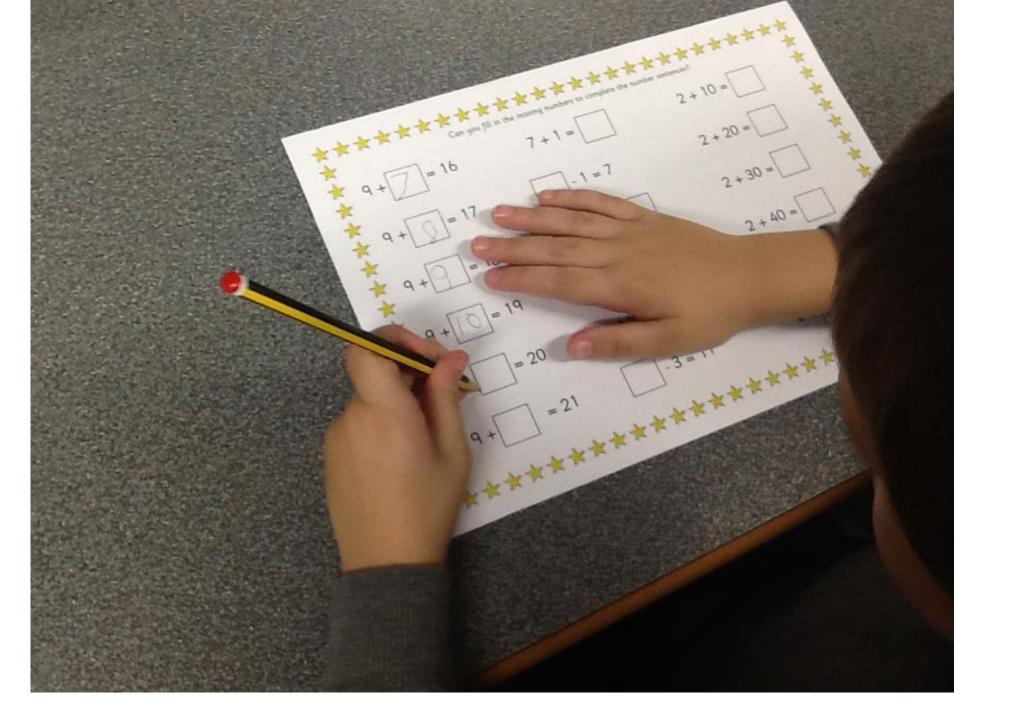


Can you fill in the missing numbers to complete the number sentences?

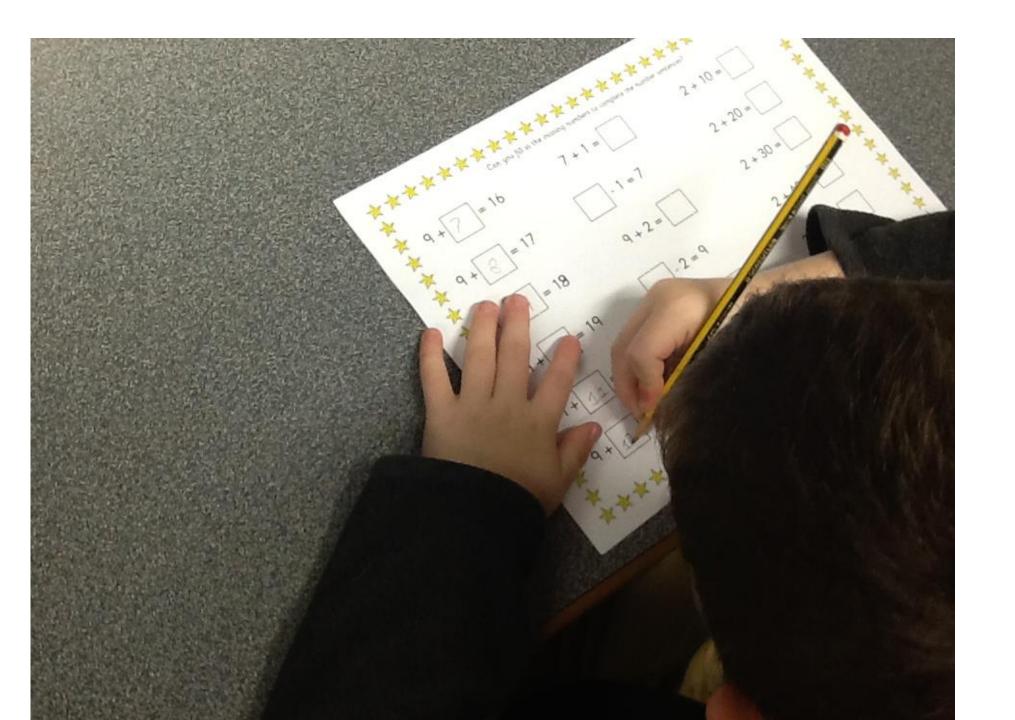




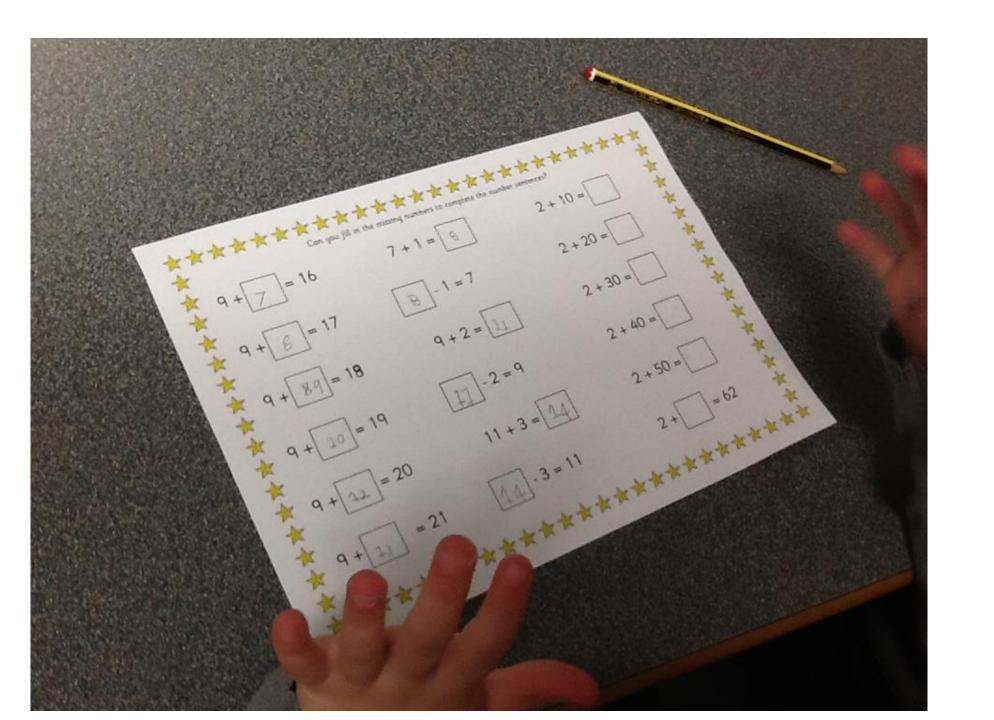




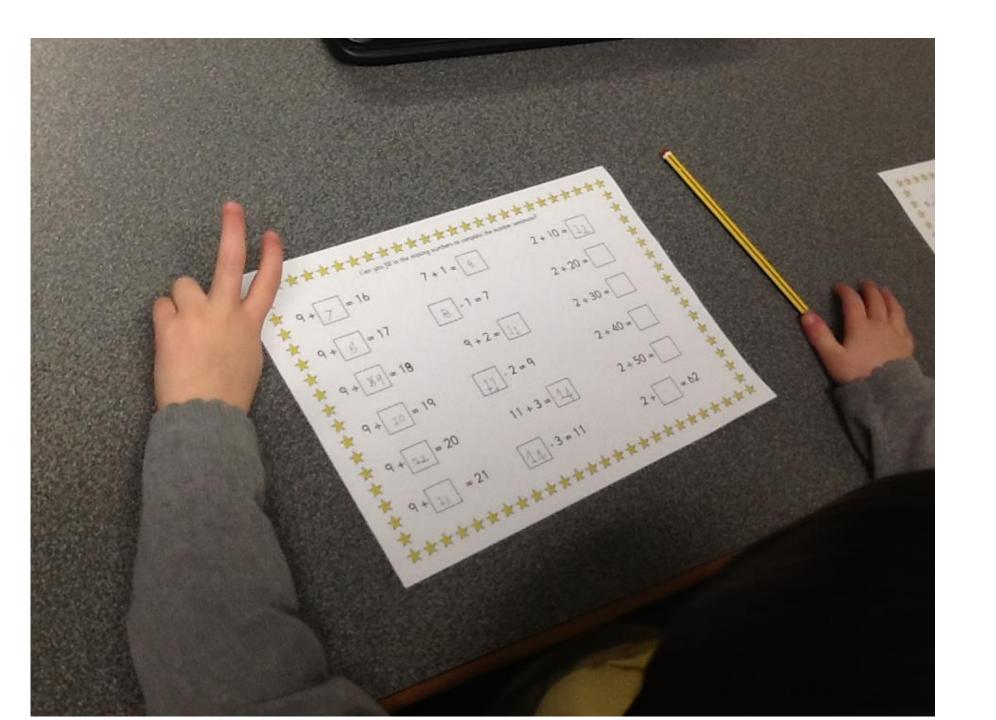




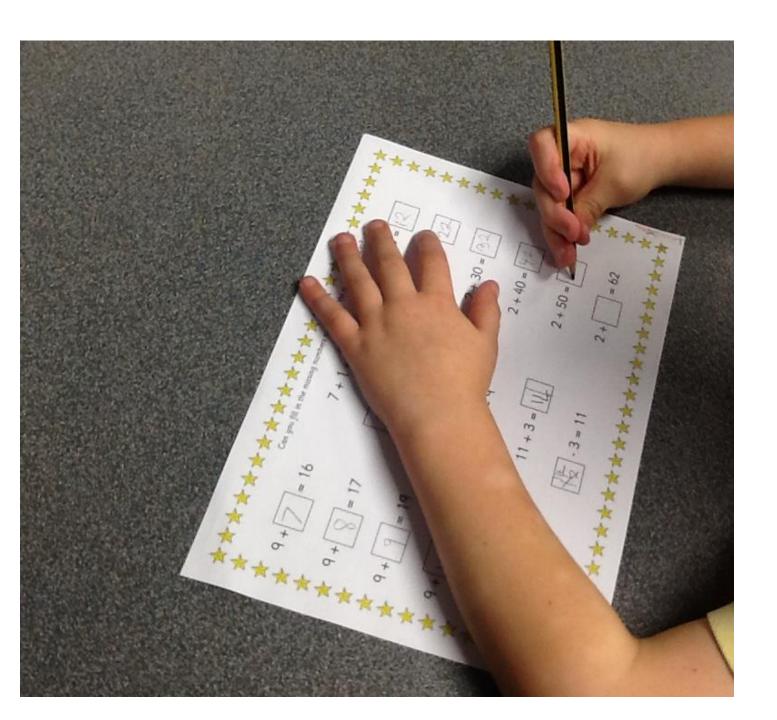














Variation examples Fluency project Enigma Maths Hub Giles Brook Primary School Claire Dinsey Year 1



Year 1



Year 1

$$2 + 3 = 5$$

I also know:



Year 1



Variation examples Fluency project Enigma Maths Hub Priory Rise Primary School Ross Bullen Year 1



$$0 + 10 = 10$$

$$10 + 10 = 20$$

$$20 + 10 = 30$$

$$30 + 10 = 40$$

$$40 + 10 = 50$$

$$60 + 10 = 70$$

$$80 + 10 = 90$$



$$3 + 10 = 13$$

$$13 + 10 = 23$$

$$23 + 10 = 33$$

$$33 + 10 = 43$$

$$43 + 10 = 53$$

$$53 + 10 = 63$$

$$63 + 10 = 73$$

$$73 + 10 = 83$$

$$83 + 10 = 93$$



$$10 - 10 = 0$$



$$17 - 10 = 7$$



Adding 10



Adding & Subtracting 10

Adding & Subtracting 10

Variation examples Fluency project Enigma Maths Hub Kents Hill School Sharon Pace Year 2



29: To continue a pattern 3 + 1 = What do you notice? 4 + 1 = What is the same? What is different?

What number sentence would come next? Continue the pattern.

	Explain why you have chosen these
	number sentences.



L9: To continue a pattern

What do you notice?

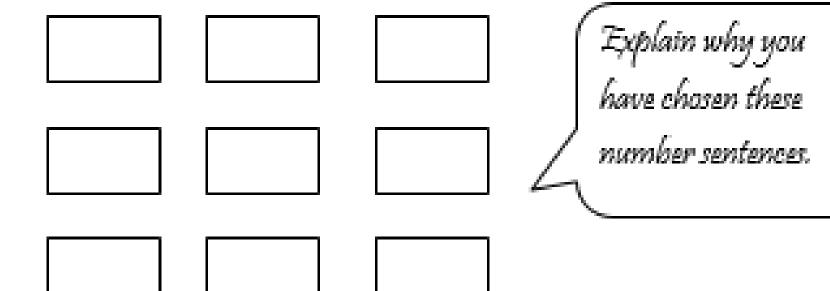
What is the same?

What is different?

What number sentence would come next? Continue the pattern.



What number sentence would come next? Continue the pattern.





Variation examples Fluency project Enigma Maths Hub Great Linford Primary School Steph Scott Year 2



<u>3.12.15</u>

OL: Can I add multiples of 10 to a number?



<u>13.11.15</u>

OL: Can I add multíples of 10 to a number?



<u> 13.11.15</u>

OL: Can I add multiples of 10 to a number?



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Examples

What do you notice?

$$127 + 10 = 137$$

$$137 + 10 = 147$$

$$147 + 10 = 157$$

$$157 + 10 = 167$$

$$167 + 10 = 177$$

Can you explain what it happening? Why?

Can you continue the pattern?



What about?



What do you notice?

$$15 - 6 = 9$$

$$25 - 6 = 19$$

$$35 - 6 = 29$$

$$35 - 16 = 19$$

$$105 - 76 = 29$$

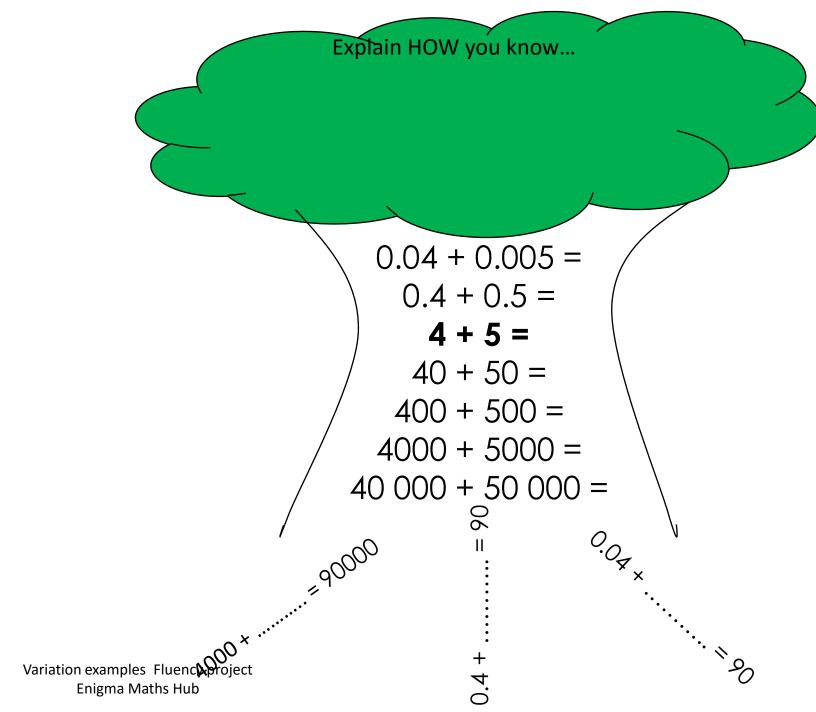
Why is that?

How could we use this to help us with other subtractions like 35 - 17 or 46 - 18?

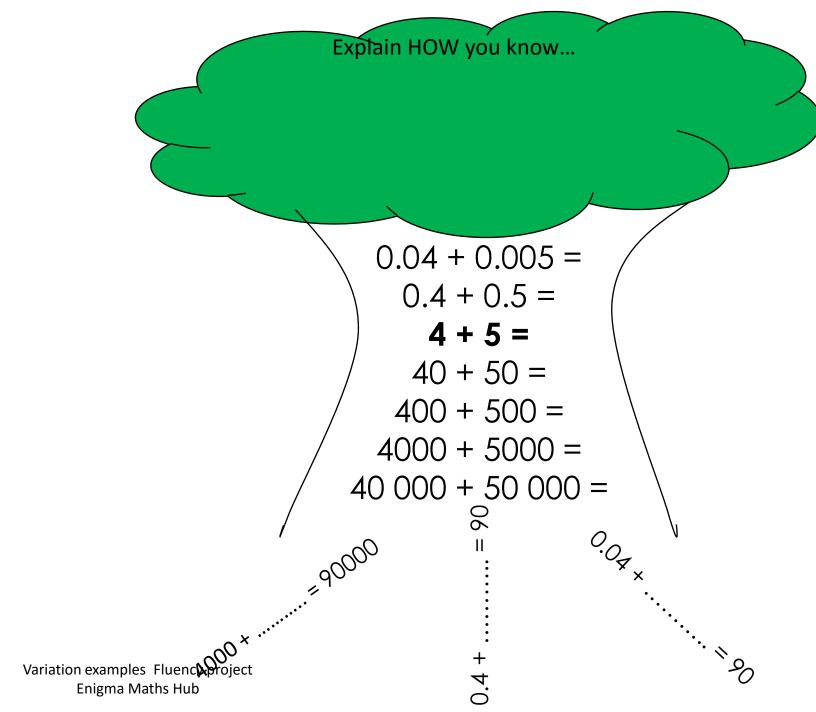


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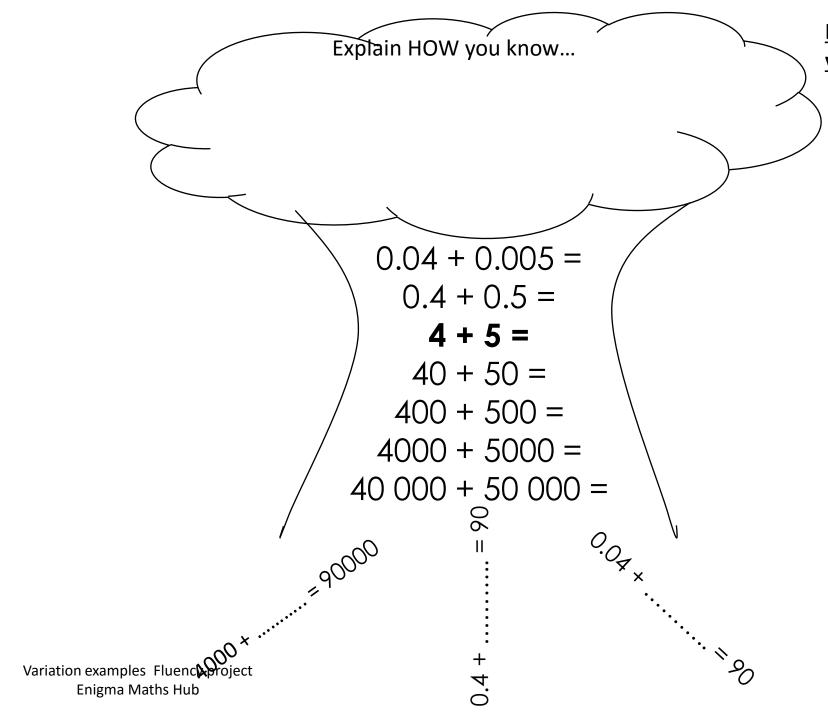




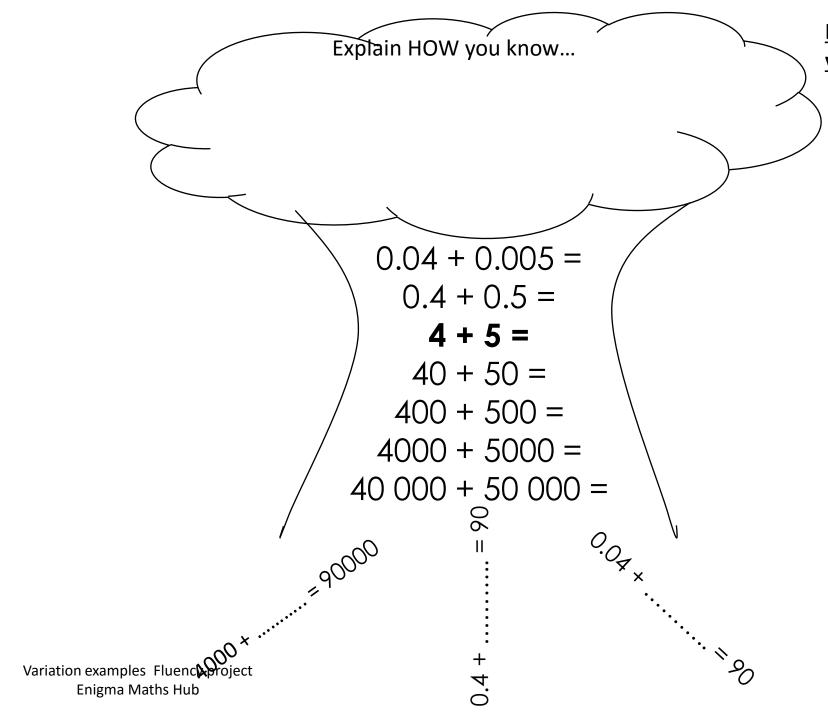














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Year 6 Variation questions

$$7.38 + 2.54 =$$

$$17.38 + 2.54 =$$

$$7.38 + 12.54 =$$

$$9.62 + 6.14 =$$

$$11.62 + 6.14 =$$

$$8.73 - 5.27 =$$

$$7.73 - 5.27 =$$

$$6.73 - 5.27 =$$

$$15.31 - 7.28 =$$

Year 6 Variation questions

Multiplication and division variation examples



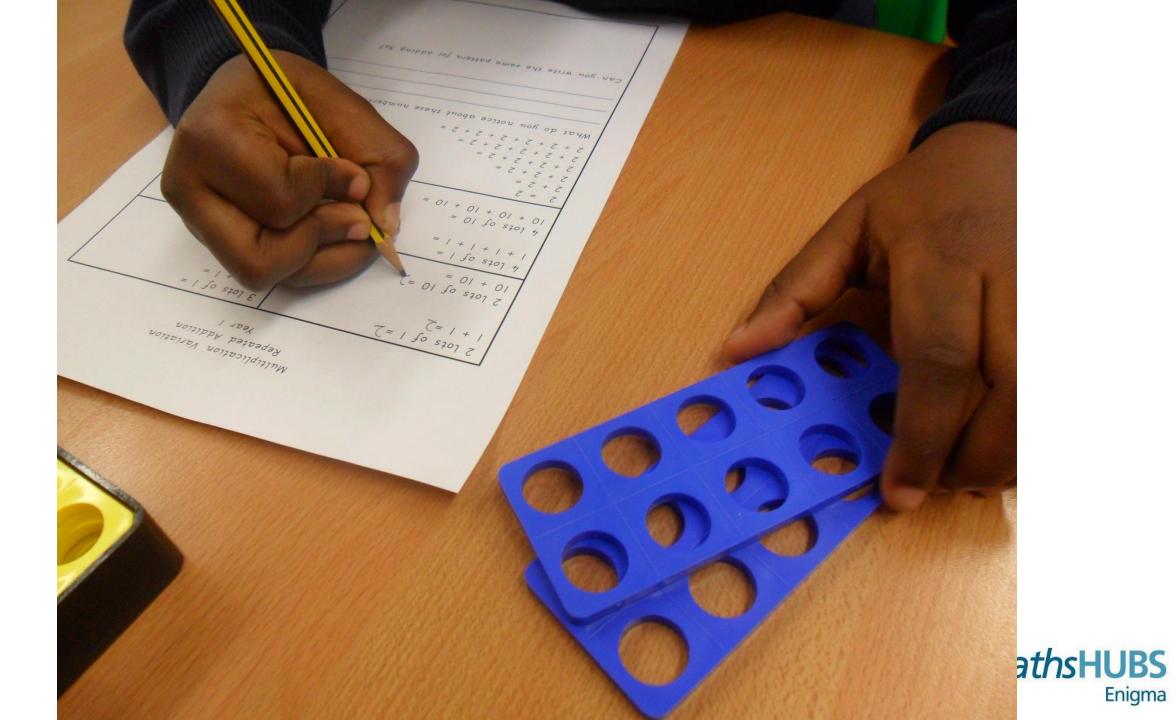
Variation examples Fluency project Enigma Maths Hub Falconhurst Primary School Laura Crawley Year 1

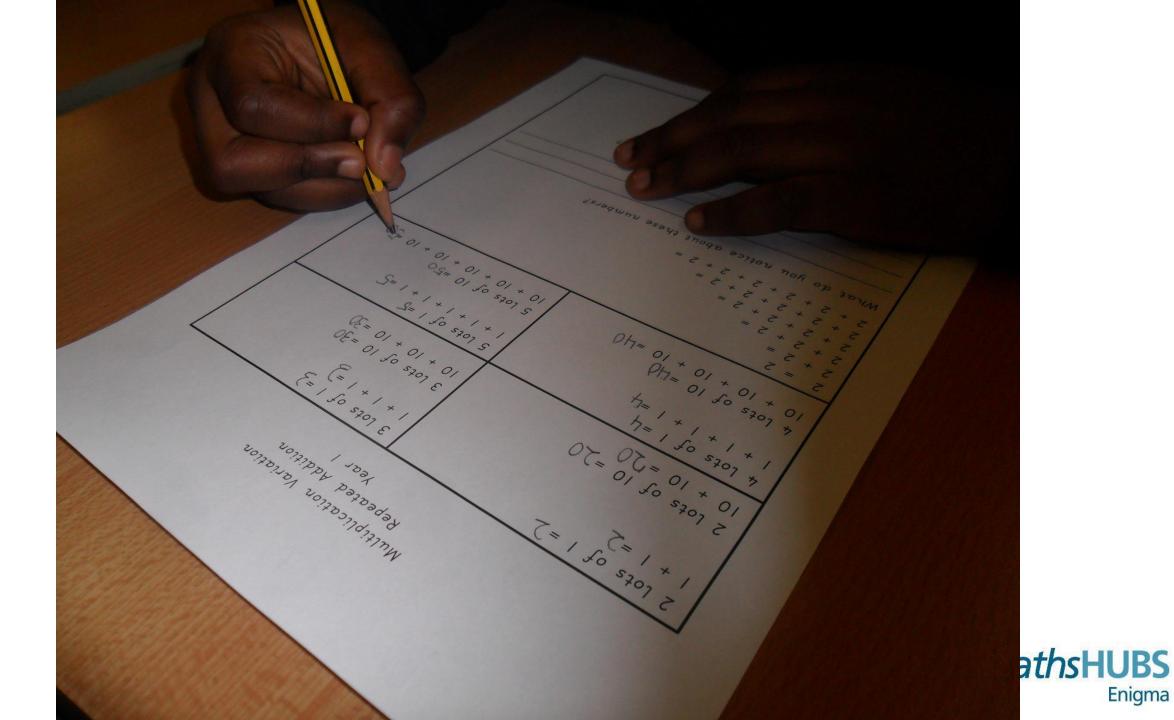


Multiplication Variation Repeated Addition Year I

2 lots of - + -	3 lots of - + + -	
2 lots of 10 10 + 10 =	3 Lots of 10 = 10 + 10 + 10 =	
4 lots of - + + + -	5 lots of = + + + + =	
4 lots of 10 = 10 + 10 + 10 + 10 =	5 lots of 10 = 10 + 10 + 10 + 10 =	
2 - 2 2 + 2 - 2 + 2 + 2 - 2 + 2 + 2 + 2 - 2 + 2 + 2 + 2 + 2 - 2 + 2 + 2 + 2 + 2 + 2 - What do you notice about these numbers?		
Can you write the same pattern for adding 5s?		







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Multiplication Variation Repeated Addition Year I

2 lots of 1 = 2 1 + 1 = 2	3 lots of = }
2 lots of 10 = 20	3 lots of $10 = 30$
10 + 10 = 20	10 + 10 + 10 = 30
4 lots of 1 = 4	5 lots of 1 = 5
1 + 1 + 1 + 1 = 4	1 + 1 + 1 + 1 + 1 = 5
4 Lots of 10 = 40	5 Lots of 10 = 50
10 + 10 + 10 + 10 = 40	10 + 10 + 10 + 10 = 50

$$2 = 2$$

$$2 + 2 = 4$$

$$2 + 2 + 2 = 6$$

$$2 + 2 + 2 + 2 + 2 = 6$$

$$2 + 2 + 2 + 2 + 2 + 2 = 6$$

$$2 + 2 + 2 + 2 + 2 + 2 = 6$$

What do you notice about these numbers?

They go up in twas

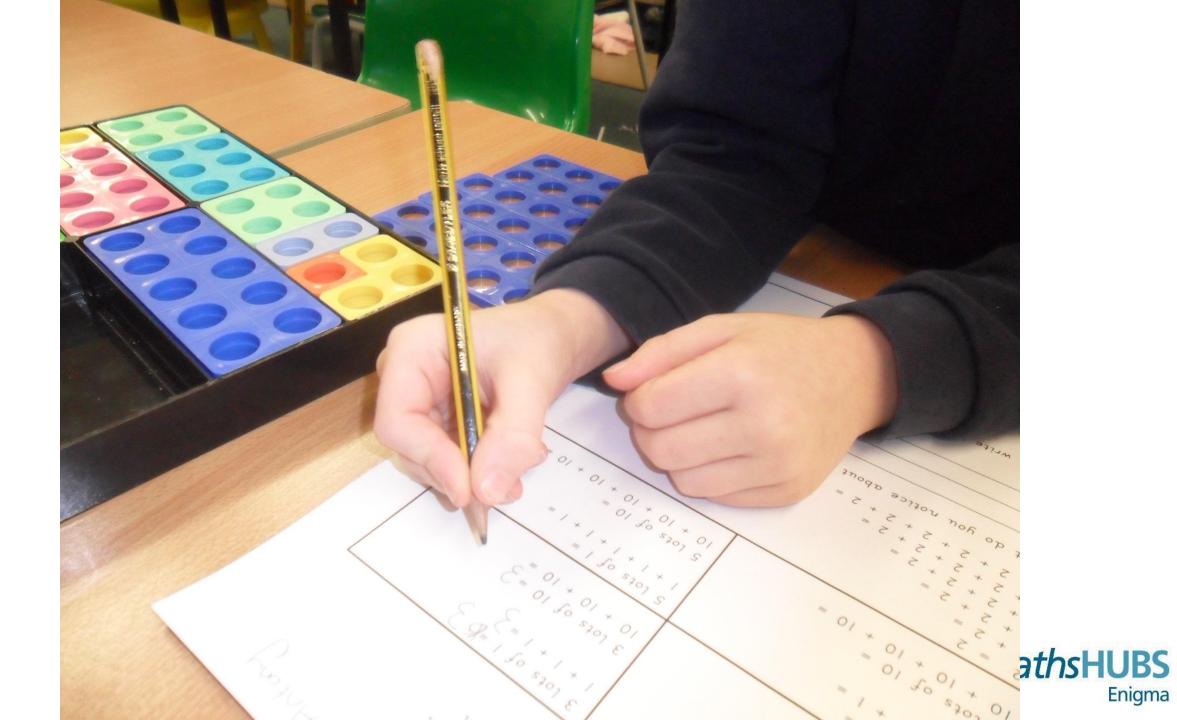
Can you write the same pattern for adding 5s?

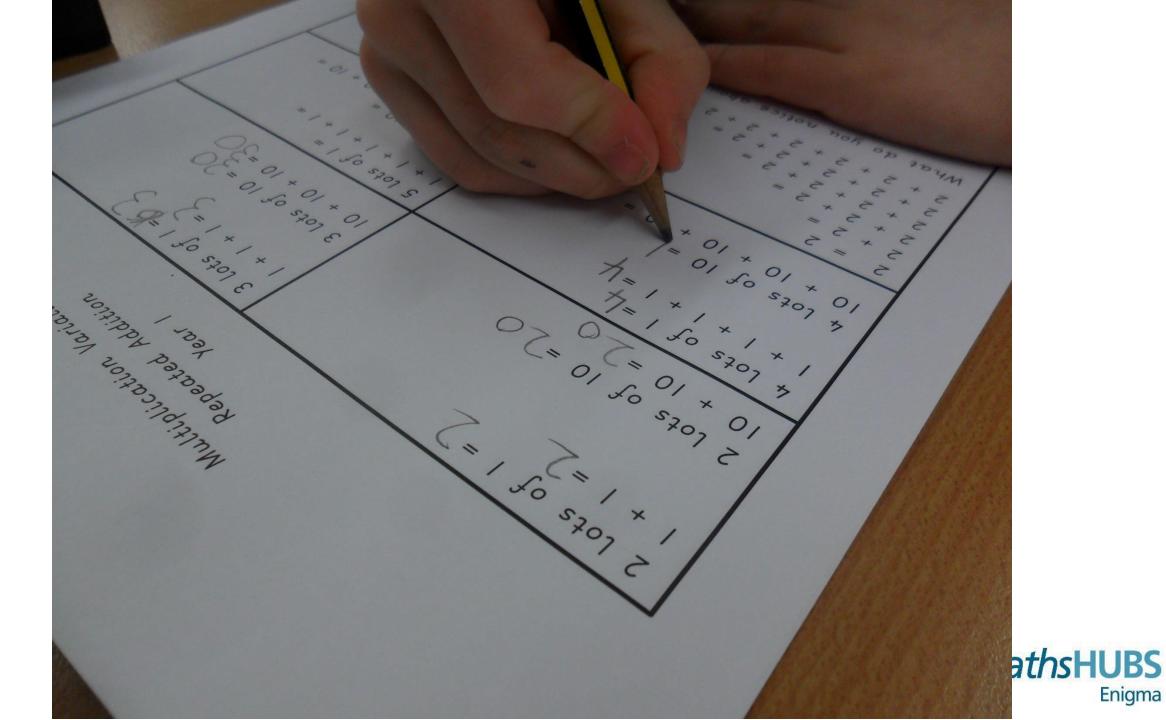
5-1

5-1

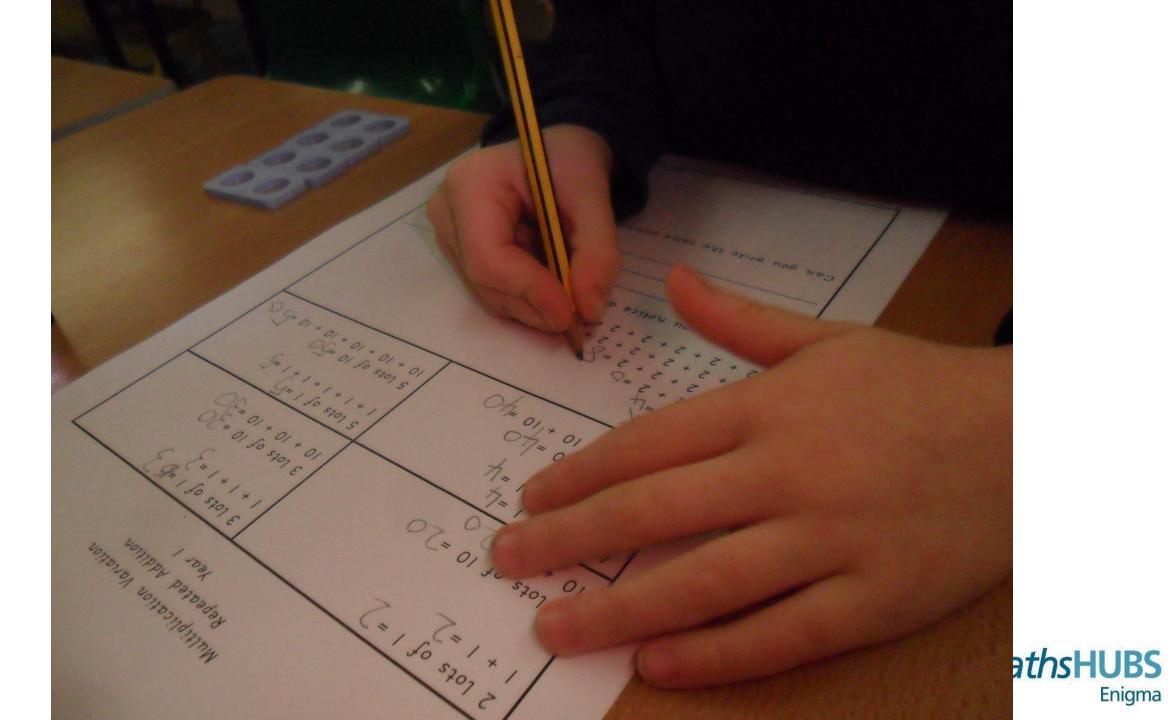








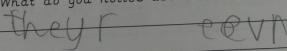
Enigma



Multiplication Variation Repeated Addition Year I

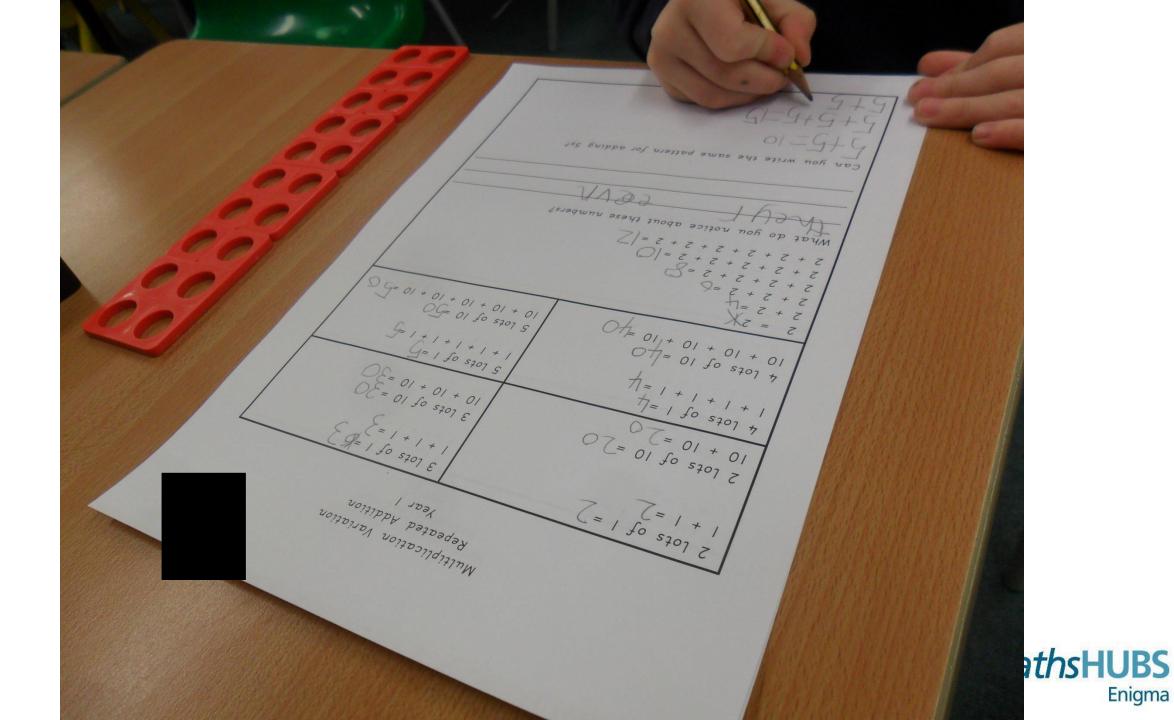
2 lots of 1 = 2 1 + 1 = 2	3 lots of 1 = 16 }
2 lots of 10 = 20	3 lots of 10 = 30
10 + 10 = 20	10 + 10 + 10 = 30
4 lots of 1 = 4	5 Lots of = 5
1 + 1 + 1 + 1 = 4	+ + + + = 5
4 Lots of 10 = 40	5 lots of 10 = 50
10 + 10 + 10 + 10 = 40	10 + 10 + 10 + 10 = 5
2 = 2V	

What do you notice about these numbers?

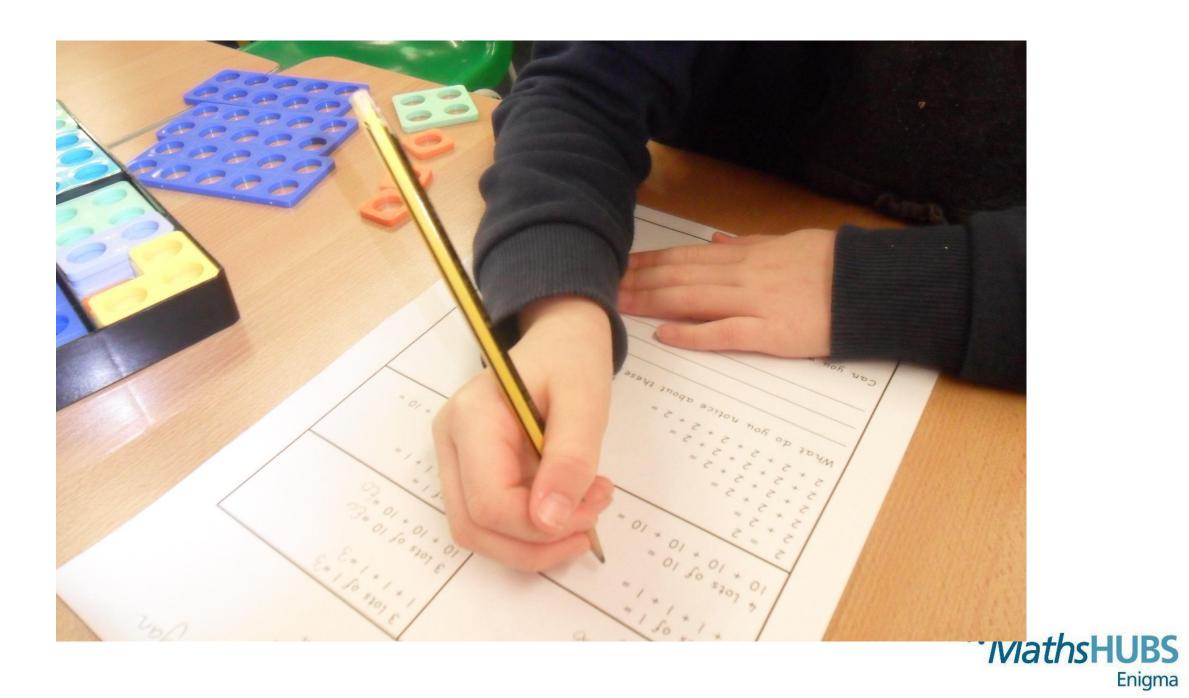


Can you write the same pattern for adding 5s?





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Multiplication Variation Repeated Addition Year I

2 lots of 1 = 2	3 lots of 1 = 3
1 + 1 = 2	1 + 1 + 1 = 3
2 lots of 10 = 5	3 lots of 10 = E0
10 + 10 = 5	10 + 10 + 10 = E0
4 lots of 1 = 4	5 Lots of 1 = 5
1 + 1 + 1 + 1 = 4	1 + 1 + 1 + 1 = 7
4 lots of 10 = 40	5 lots of $10 = 50$
10 + 10 + 10 + 10 = 46	10 + 10 + 10 + 10 = 50

 $2 = 2 \\
2 + 2 = 50 \\
2 + 2 + 2 = 2 \\
2 + 2 + 2 + 2 + 2 = 50 \\
2 + 2 + 2 + 2 + 2 + 2 = 50$ 2 + 2 + 2 + 2 + 2 + 2 = 50

What do you notice about these numbers?

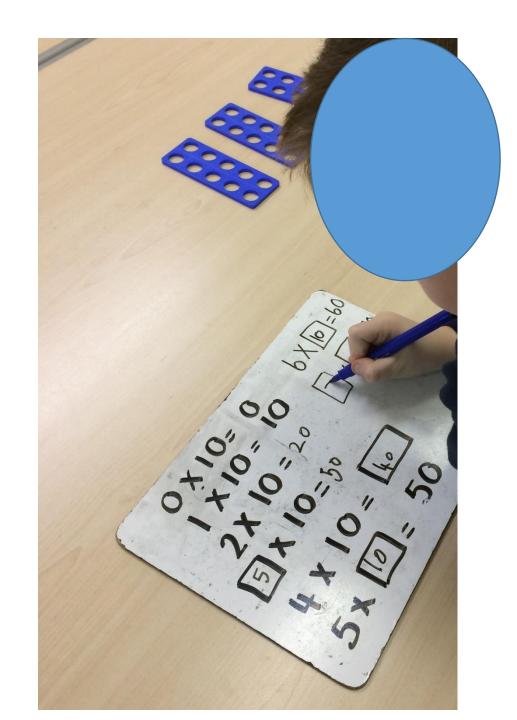
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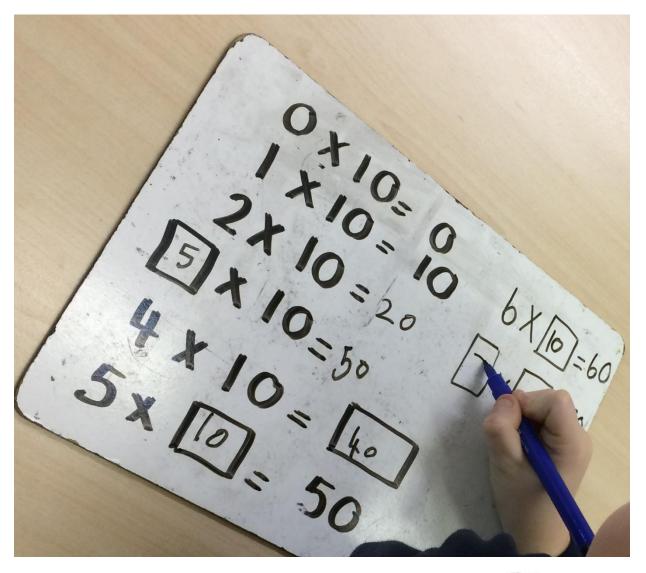
Can you write the same pattern for adding 5s?



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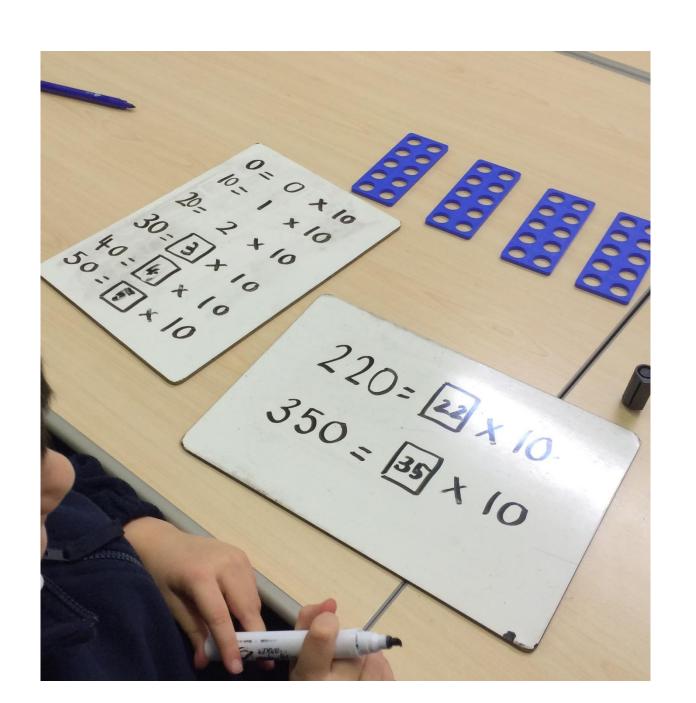














Variation examples Fluency project Enigma Maths Hub Priory Rise Primary School Ross Bullen Year 1



$$2 + 2 = 4$$

$$2 + 2 + 2 = 4$$

$$2 + 2 + 2 + 2 = 8$$

$$2 + 2 + 2 + 2 + 2 + 2 = 10$$

$$2 + 2 + 2 + 2 + 2 + 2 + 2 = 10$$

$$2 + 2 + 2 + 2 + 2 + 2 + 2 = 14$$

$$2 + 2 + 2 + 2 + 2 + 2 + 2 = 16$$

$$2 + 2 + 2 + 2 + 2 + 2 + 2 = 16$$

= 10 + = = 15 + + 5 + + + +



Variation examples Fluency project Enigma Maths Hub Kents Hill School Sharon Pace Year 2



LI: To use repeated addition

What do you notice?

What is the same?

What is different?



LI: To continue a pattern

What jottings could you make?

Can you finish the last number sentence by following the pattern?

Variation examples Fluency project Enigma Maths Hub Great Linford Primary School Steph Scott Year 2





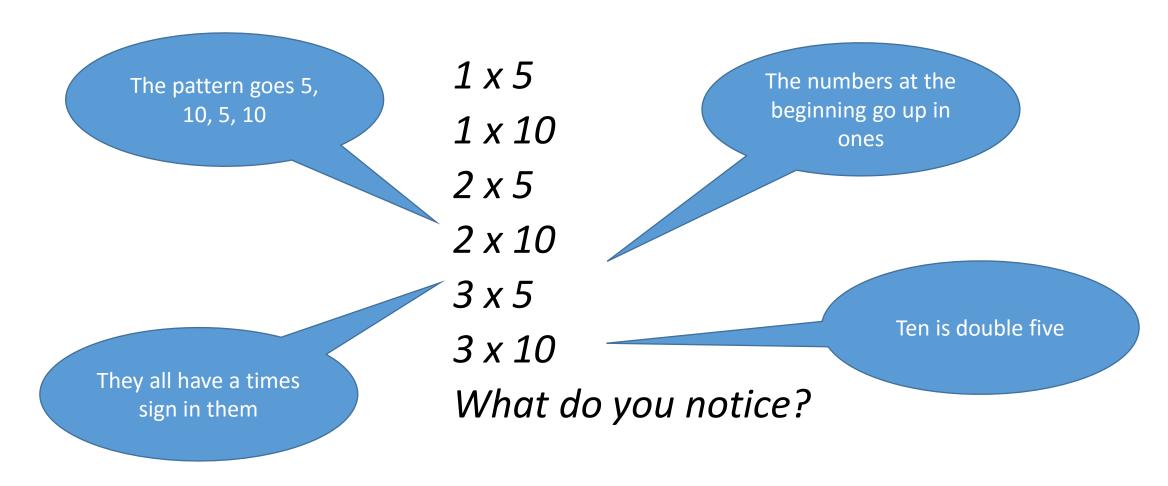
2+2+2+2+2=12 +++++=2



Variation examples Fluency project Enigma Maths Hub Clare Pickering Year 2 Trinity C of E Primary School Aldwincle, Northants



I began by displaying the following for oral discussion:



We then focused on finding the products for each calculation and discussed further how the products of the 10x table were double the 5x table.

$$1 \times 5 = 5$$

$$1 \times 10 = 10$$

$$2 \times 5 = 10$$

$$2 \times 10 = 20$$

$$3 \times 5 = 15$$

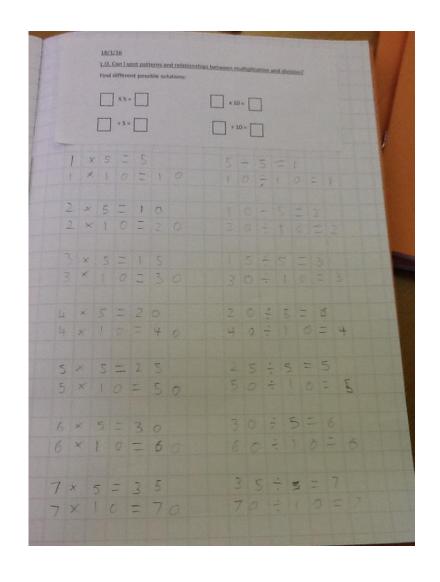
$$3 \times 10 = 30$$

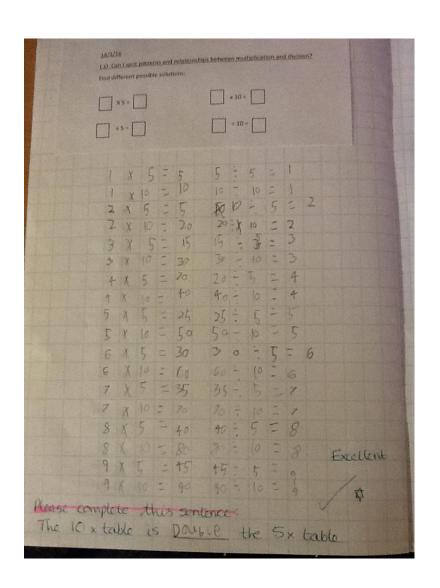
I then posed the question: What if I wrote:

$$\Box \div 5 = 2$$

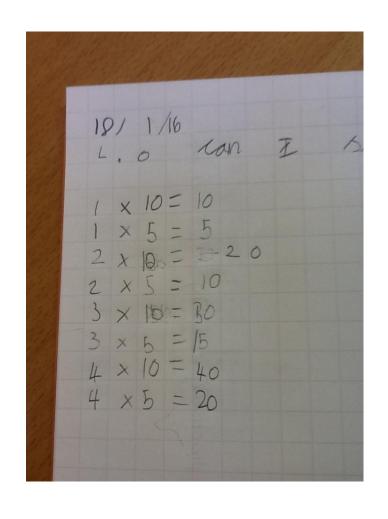
Children were given the following in their books to find different possible solutions:

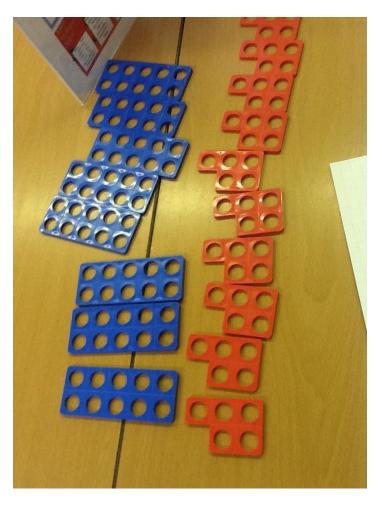
Examples of children's work:





LA children focused on spotting patterns in the 5 and 10 times tables without division:





Variation examples Fluency project Enigma Maths Hub Caroline Haslett Primary School Louise Cullen Year 3 example



$$2 \times 4 = 2 \times 8 =$$

$$2 \times 40 = 2 \times 80 =$$

$$2 \times 400 = 2 \times \boxed{} = 1600$$

Year 3: What do you notice?

- "They all have times in them."
- "The four gets bigger, then the two."
- "There are fours on one side and twos on the other."
- "The fours are going 4, 40, 400."



"Can you use the 2 x 4 column to work out the 2 x 8 column?"

Some responses

- "No because they are different numbers."
- "The second one has an 80. It goes in a row so you have to put another zero on."
- "Instead of the 4's it's the 8's."

The children found it very difficult to see the link and only one child was able to say: "8 is double 4."

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Use your place value chart to help you complete the following on a whiteboard:

$$370 \div 1 = 370$$

$$370 \div 10 = 37$$

$$150 \div 1 = 150$$

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What do you notice?

the answer is the same num

True or false?

$$40 \div 10 = 4 \checkmark 4 \times 10 = 40$$

 $40 \div 1 = 4 \times 40 \div 1 = 40$ or $40 \div 10 = 4$
 $400 \div 10 = 4 \times 400 \div 10 = 40$
 $400 \div 100 = 4 \checkmark 4 \times 100 = 400$

How do you know?





Complete:

$$1 \times 9 = 9$$
 $9 \times 1 = 9$

$$1 \times 0 = 0$$
 $0 \times 1 = 0$

$$0 \times 9 = 0$$
 $9 \times 0 = 0$

$$8 \times 1 = 8 \qquad 8 \times 0 = 0$$

$$1 \times 1 = 1 \times 2 = 2$$



What do you notice?

$$2 \times 3 = 6$$

$$2 \times 30 = 60$$

$$20 \times 3 = 60$$

$$2 \times 300 = 600$$

$$20 \times 30 = 600$$

$$4 \times 5 = 20$$

$$4 \times 50 = 200$$

$$40 \times 5 = 200$$

$$4 \times 500 = 2000$$

$$40 \times 50 = 2000$$



What do you notice?

$$2 \times 4 = 8$$

$$6 \times 4 = 24$$

$$10 \times 4 = 40$$

$$12 \times 4 = 48$$

$$2 \times 8 = 16$$

$$6 \times 8 = 48$$

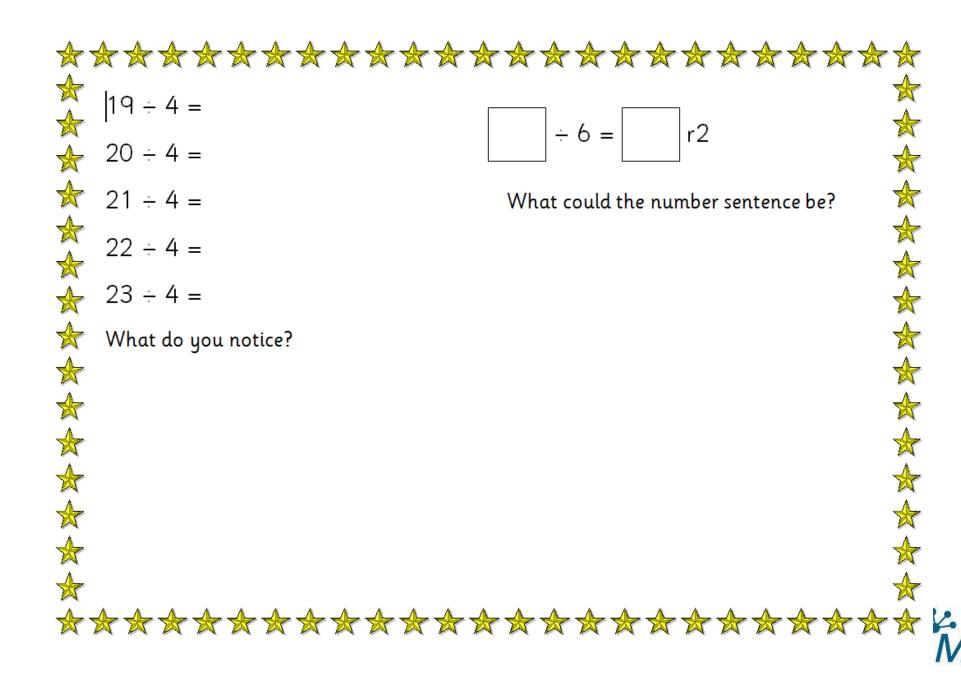
$$10 \times 8 = 80$$

$$12 \times 8 = 96$$

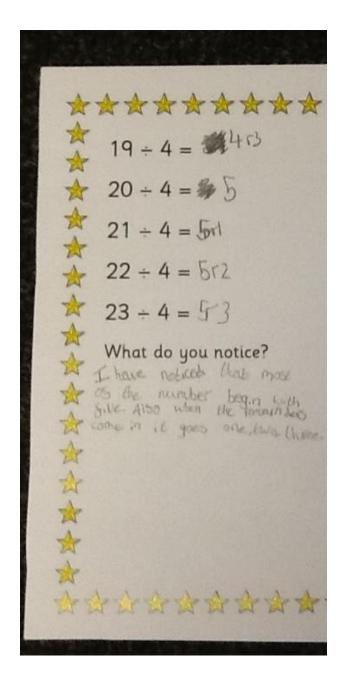


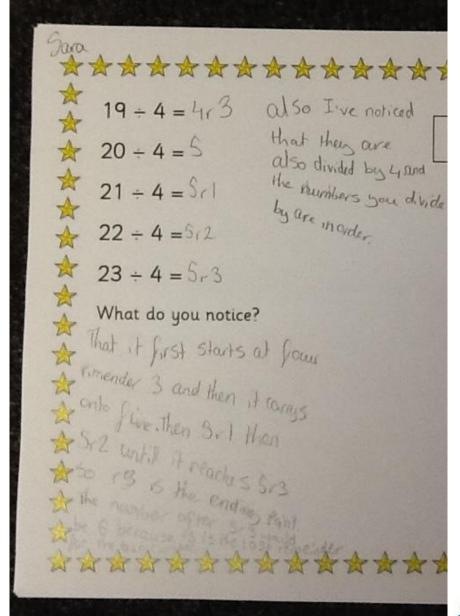
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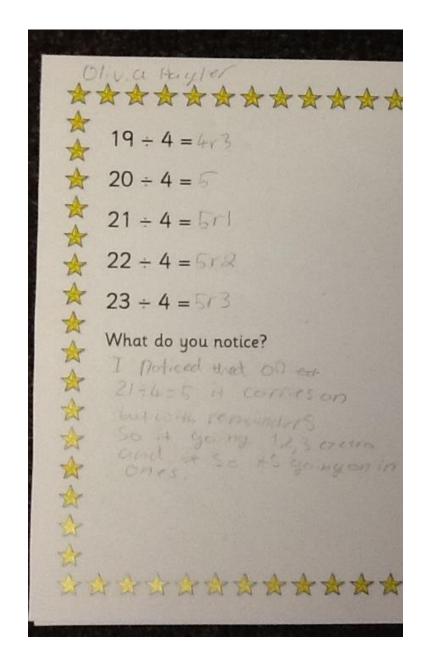


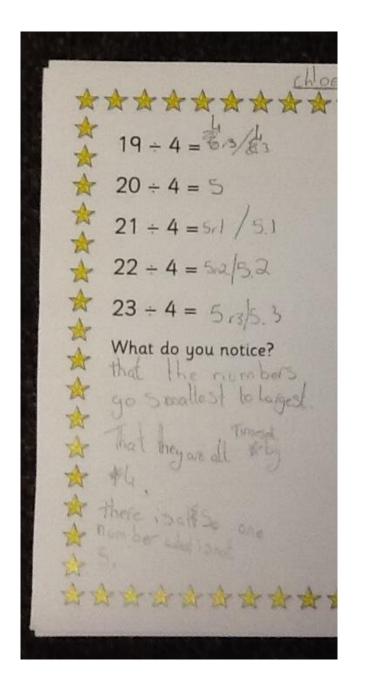
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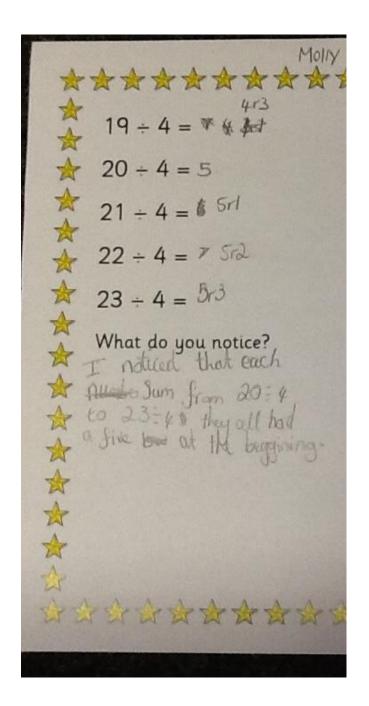


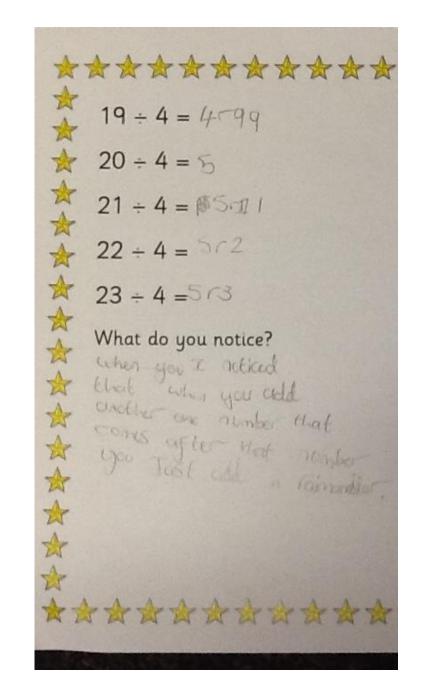




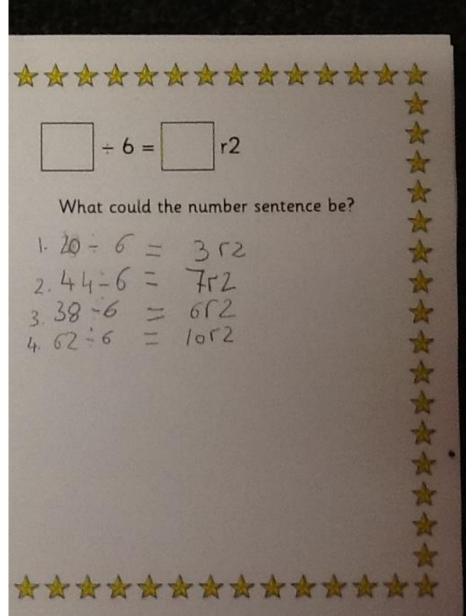


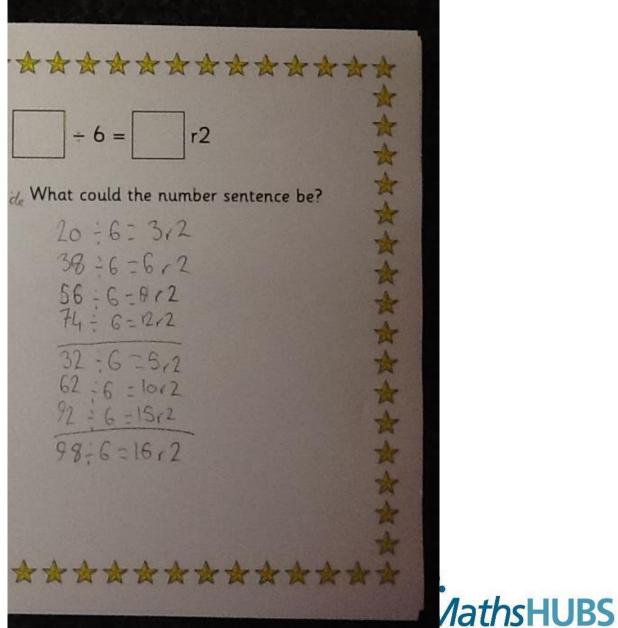




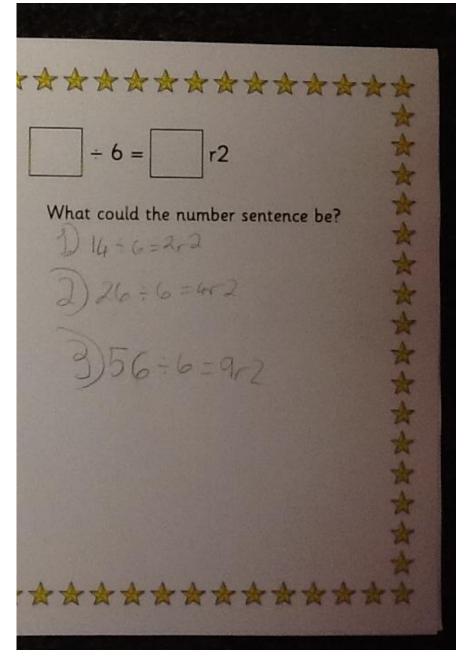


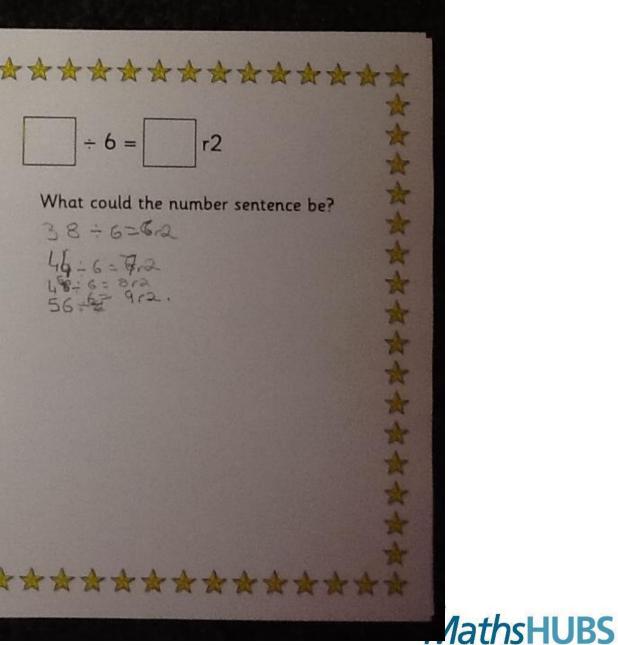


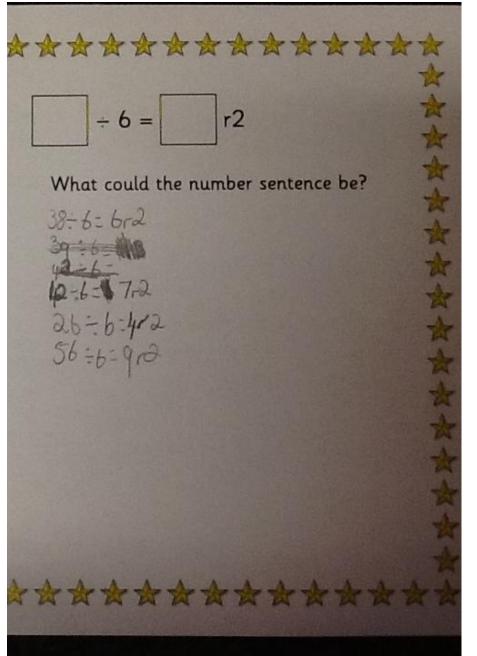


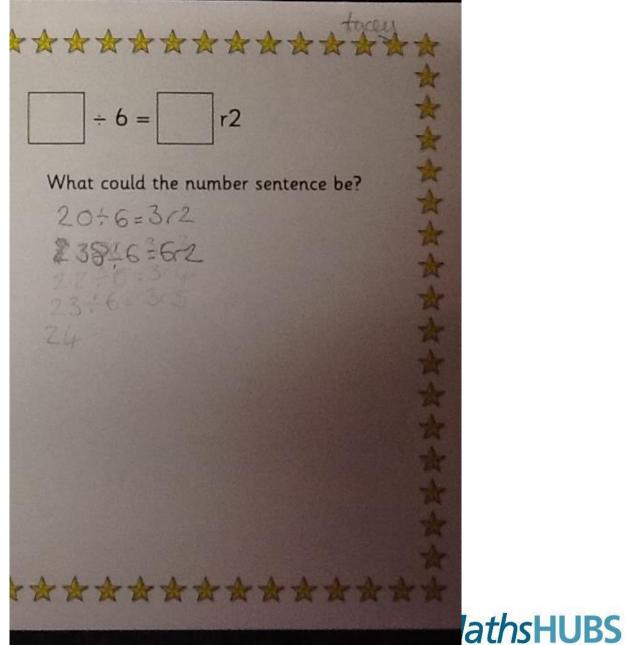


Enigma









Enigma

Variation examples Fluency project Enigma Maths Hub Spring Lane School Laura Butler Emilie Harbottle Year 5



LO: To multiply by multiples of 10

1)If
$$5 \times 3 = 15$$
 then $50 \times 3 =$ and $500 \times 3 =$

2)If
$$5 \times 4 = 20$$
 then $50 \times 4 =$ and $500 \times 4 =$

3) If
$$5 \times 5 = 25$$
 then $50 \times 5 = ...$ and $500 \times 5 = ...$

4) If
$$5 \times 6 = 30$$
 then $50 \times 6 =$ and $50 \times 60 = ...$

5) If
$$8 \times 3 = 24$$
 then $3 \times 80 =$ and $800 \times 3 =$



(Be careful with the next one!)

1) If
$$9 \times 3 = 27$$
 then $90 \times 30 =$ and $900 \times 30 = ...$

2) If
$$4 \times 9 = 36$$
 then $40 \times 9 =$ and $400 \times 9 = ...$

3) If
$$8 \times 8 = 64$$
 then $80 \times 8 =$ and $800 \times 8 =$

4) If
$$9 \times 8 = 72$$
 then $90 \times 8 =$ and $900 \times 8 =$

5)If
$$6 \times 9 = 54$$
 then $60 \times 9 =$ and $600 \times 9 =$



Variation examples Fluency project Enigma Maths Hub Loughton School Cathy Woodward Year 6



$$19 \div 4 =$$

$$20 \div 4 =$$

$$21 \div 4 =$$

$$22 \div 4 =$$

$$23 \div 4 =$$

$$24 \div 4 =$$

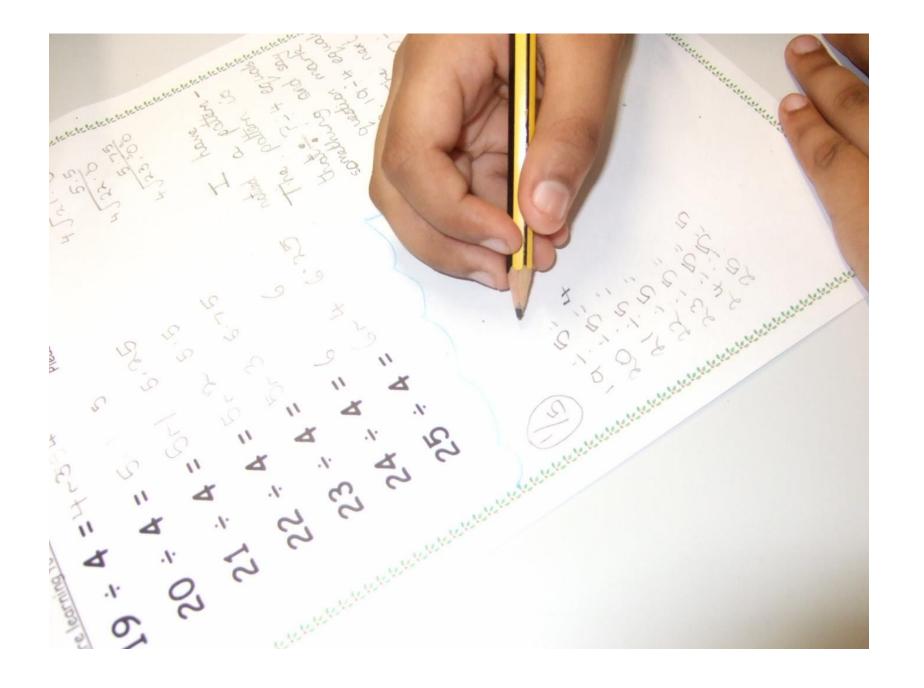
$$25 \div 4 =$$

Do you notice a pattern?

Please explain.

Would the answer increase by 1/5 each time, if you divide by 5?











16.12.15 Trisha We are learning to investigate dividing by 4 4 = 4r3 = 4.75 = Do you notice a pattern? Please explain. I have noticed a pattern. I have noticed that the remainders go up 22 ÷ 4 = 5+2 = 5.5 = 51 to remainder 3 because it's the $23 \div 4 = 5 \div 3 = 5.75 = 5.75 = 5.75 = 6 = 6$ 4 S. I have also noticed that the





We hope that these were useful

