



Developing Mastery in Mathematical fluency

Teaching for Mastery

- Access
- Pattern
- Making Connections

**Representation
& Structure**

- Chains of Reasoning
- Making Connections

**Mathematical
Thinking**

Coherence

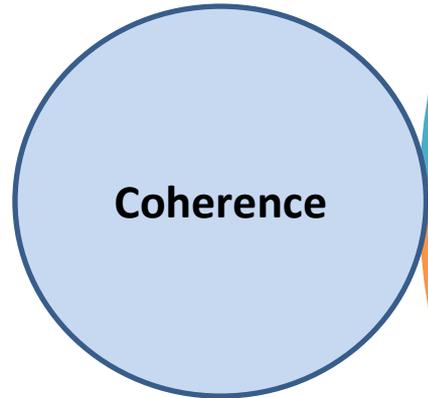
Small steps are easier to take

Variation

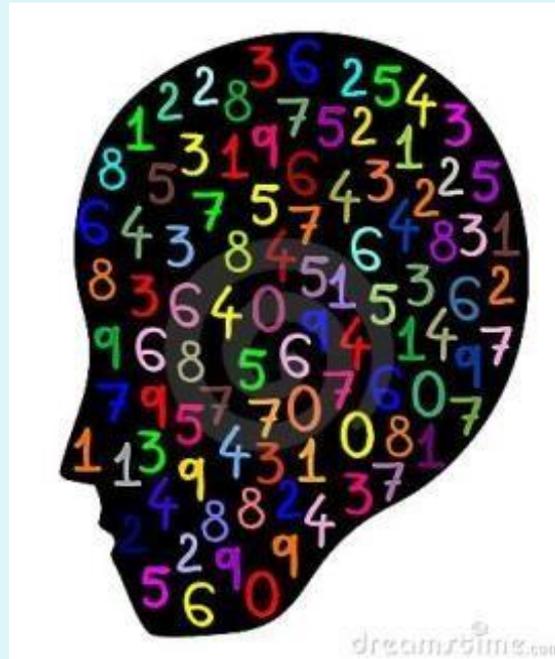
- Procedural
- Conceptual
- Making Connections

Fluency

- Number Facts
- Table Facts
- Making Connections



Developing mastery in fluency



A Look at Reading Fluency...

Fluency in reading is important because it provides a bridge between word recognition and comprehension. **Because fluent readers do not have to concentrate on decoding words, they can focus their attention on what the text means.** They can make connections among the ideas in the text and between the text and their background knowledge. In other words, **fluent readers recognize words and comprehend at the same time. Less fluent readers, however, must focus their attention on figuring out the words, leaving them little attention for understanding the text.**

Institute for Literacy. Put Reading First – K-3.

http://www.nifl.gov/partnershipforreading/publications/reading_first1fluency.html

Tracking recall of Multiplication Facts

x	2	3	4	5	6	7	8	9	11	12
2	4	6	8	10	12	14	16	18	22	24
3		9	12	15	18	21	24	27	33	36
4			16	20	24	28	32	36	44	48
5				25	30	35	40	45	55	60
6					36	42	48	54	66	72
7						49	56	63	77	84
8							64	72	88	96
9								81	99	108
11									121	132
12										144

19 facts to learn in Y2

21 facts to learn in Year 3

15 Facts to learn in Year 4

Tracking Fluency in Addition Bonds

+	2	3	4	5	6	7	8	9
2	4	5	6	7	8	9	10	11
3		6	7	8	9	10	11	12
4			8	9	10	11	12	13
5				10	11	12	13	14
6					12	13	14	15
7						14	15	16
8							16	17
9								18

16 Facts to learn in Reception

20 Facts to learn in Year 1

Y2 – Apply to bonds within 100

1.	$4 + 8 =$	$2 + 1 =$	$9 + 1 =$
	$6 + 8 =$	$4 + 3 =$	$9 + 3 =$
	$8 + 8 =$	$6 + 5 =$	$9 + 5 =$
	$10 + 8 =$	$8 + 7 =$	$9 + 7 =$
	$12 + 8 =$	$10 + 9 =$	$9 + 9 =$

2	$37 - 20$	$64 - 10$	$46 - 10$	$55 - 10$	$72 - 10$
	$37 - 10$	$64 - 20$	$46 - 20$	$55 - 30$	$72 - 30$
	$37 - 30$	$64 - 50$	$46 - 30$	$55 - 50$	$72 - 40$

3	$64 + 30$	$45 + 20$	$81 + 10$	$72 + 20$	$33 + 30$
	$64 - 30$	$45 - 20$	$81 - 10$	$72 - 20$	$33 - 30$

4	$58 - 24 = \underline{\quad}$	$36 - 25 = \underline{\quad}$	$53 - 22 = \underline{\quad}$	$49 - 24 = \underline{\quad}$
	$57 - 25 = \underline{\quad}$	$46 - 24 = \underline{\quad}$	$64 - 23 = \underline{\quad}$	$48 - 25 = \underline{\quad}$
	$56 - 26 = \underline{\quad}$	$56 - 23 = \underline{\quad}$	$75 - 24 = \underline{\quad}$	$47 - 26 = \underline{\quad}$

What is varying (and what is not)?

$5 \times 3 =$

$5 \times 30 =$

$5 \times 300 =$

$7 \times 5 =$

$7 \times 50 =$

$7 \times 500 =$

$5 \times 7 =$

$5 \times 70 =$

$5 \times 700 =$

$3 \times 5 =$

$3 \times 50 =$

$3 \times 500 =$

2 用自己喜欢的方法计算，谁快？

$5 \times 400 =$

$8 \times 300 =$

$10 \times 700 =$

$5 \times 900 =$

$4 \times 800 =$

$9 \times 700 =$

$4 \times 900 =$

$4 \times 800 =$

$4 \times 700 =$

$7 \times 800 =$

$6 \times 800 =$

$5 \times 800 =$

$180 \div 2 =$

$180 \div 20 =$

$270 \div 30 =$

$270 \div 90 =$

$160 \div 4 =$

$160 \div 40 =$

$270 \div 3 =$

$270 \div 9 =$

$480 \div 6 =$

$480 \div 60 =$

$540 \div 90 =$

$540 \div 9 =$

$500 \div 5 =$

$500 \div 50 =$

$400 \div 80 =$

$400 \div 8 =$

Example: Round any number to the nearest 10

We want pupils to notice:

- that when rounding to the nearest multiple 10 we are particularly interested in the units digit (the bit that is not a multiple of 10)
- what is half way between consecutive multiples of 10
- how far a number is from each multiple of 10

$11 + \square = 20$

$11 - \square = 10$

$12 + \square = 20$

$12 - \square = 10$

$13 + \square = 20$

$13 - \square = 10$

$14 + \square = 20$

$14 - \square = 10$

$15 + \square = 20$

$15 - \square = 10$

$16 + \square = 20$

$16 - \square = 10$

$17 + \square = 20$

$17 - \square = 10$

$18 + \square = 20$

$18 - \square = 10$

$21 + \square = 30$

$21 - \square = 20$

$42 + \square = 50$

$42 - \square = 40$

$85 + \square = 90$

$85 - \square = 80$

$98 + \square = 100$

$98 - \square = 90$

11 Nearest
 10 →

12 Nearest
 10 →

13 Nearest
 10 →

14 Nearest
 10 →

15 Nearest
 10 →

16 Nearest
 10 →

17 Nearest
 10 →

18 Nearest
 10 →

16 Nearest
 10 →

26 Nearest
 10 →

36 Nearest
 10 →

76 Nearest
 10 →

86 Nearest
 10 →

96 Nearest
 10 →

423 Nearest
 10 →

424 Nearest
 10 →

425 Nearest
 10 →

426 Nearest
 10 →

427 Nearest
 10 →

Designing purposeful learning for mathematics

Object of learning:

$23 + 10 = 23 + 10 =$

$23 + 11 = 23 + 9 =$

$23 + 12 = 23 + 8 =$

$23 + 30 = \square$

$23 + \square = 54$

Adding a is near to 10.

$23 + 20 = 23 + 20 = 23 + 30 = \square$

$23 + 21 = 23 + 19 = 23 + \square = 52$

$23 + 22 = 23 + 18 = 23 + \square = 51$

$23 + 30 = 23 + 30 = 53 + \square = 93$

$23 + 31 = 23 + 29 = 53 + \square = 92$

$23 + 32 = 23 + 28 = 53 + \square = 91$

Designing purposeful learning for mathematics

$23 + 10 =$

$23 + 11 =$

$23 + 12 =$

$23 + 20 =$

$23 + 21 =$

$23 + 22 =$

$23 + 30 =$

$23 + 31 =$

$23 + 32 =$

$23 + 10 =$

$23 + 9 =$

$23 + 8 =$

$23 + 20 =$

$23 + 19 =$

$23 + 18 =$

$23 + 30 =$

$23 + 29 =$

$23 + 28 =$

$23 + 30 = \square$

$23 + \square = 54$

$23 + 30 = \square$

$23 + \square = 52$

$23 + \square = 51$

$53 + \square = 93$

$53 + \square = 92$

$53 + \square = 91$

What do you notice?

19	20	39
12	2	14
31	22	

7

9

13

16

10

15

		24
	13	25
31	18	

Learning facts: effective teaching strategies

- **Make connections** between facts – related facts are easier to learn than unrelated ones.
- **Understand why** – e.g. why any number $\times 0$ is 0 and any number $\times 1$ is itself
- **Reasoning about answers** – $3 + 4$ cannot equal 6 since $3 + 3 = 6$
- **Making time for practice**

Develop flexibility

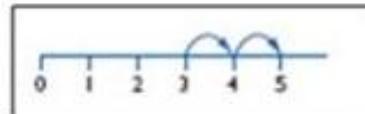
(Baroody 2006)

Resources to help build concepts

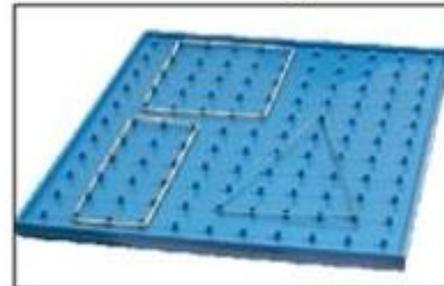
Numicon



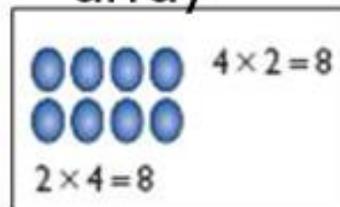
number line



geoboard



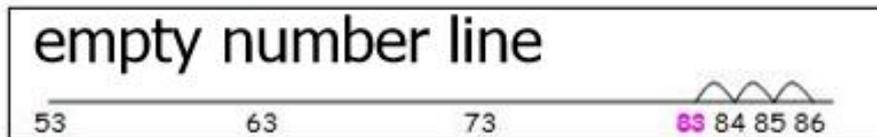
array



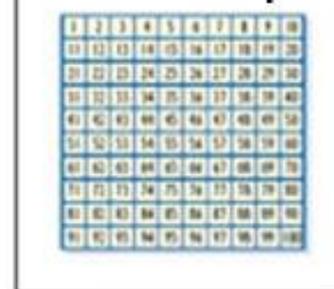
counting stick or metre rule



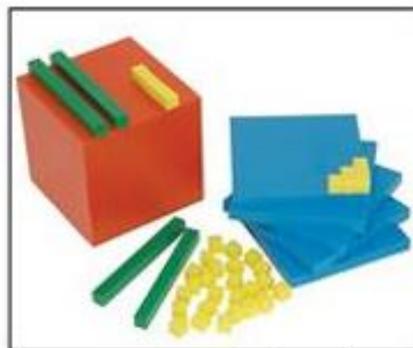
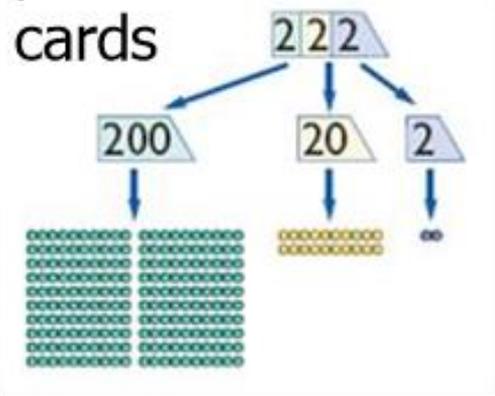
empty number line



hundred square

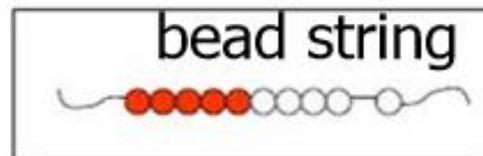


place value



Dienes blocks
base-ten blocks

bead string



Models, images and practical apparatus

..... play an important part in supporting pupils' conceptual understanding and reasoning

A Look at Numerical Fluency

Fluency in Mathematics is important because it provides a **bridge between number recognition and problem solving comprehension**. Because people who are numerically fluent do not have to concentrate on operation facts, they can focus their attention on what the problem means. They can make connections among the ideas in the problem and their background knowledge. In other words, **people who are numerically fluent recognize how to compose and decompose numbers based on patterns and comprehend how to use those numerical patterns to solve problems**. People who are less fluent, however, must focus their attention on the operations, leaving them little attention for **understanding the problem**.