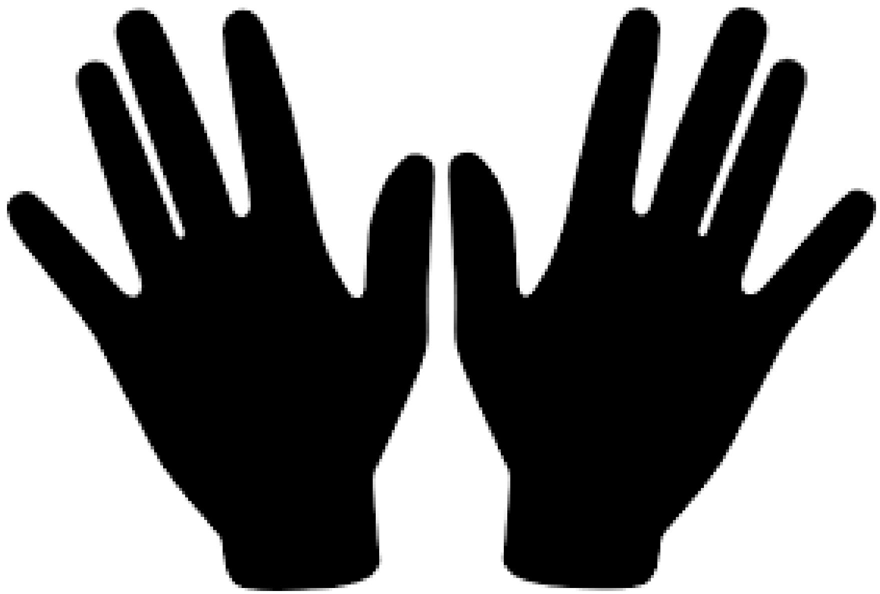
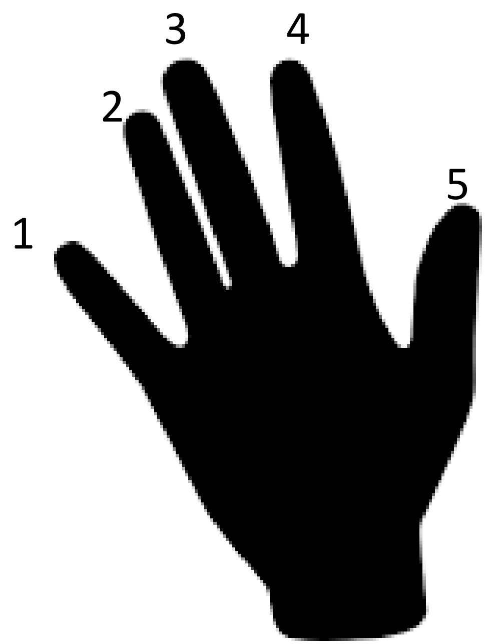
The Secrets of Shanghai Mathematics Teaching

Necessary Aspects of Learning

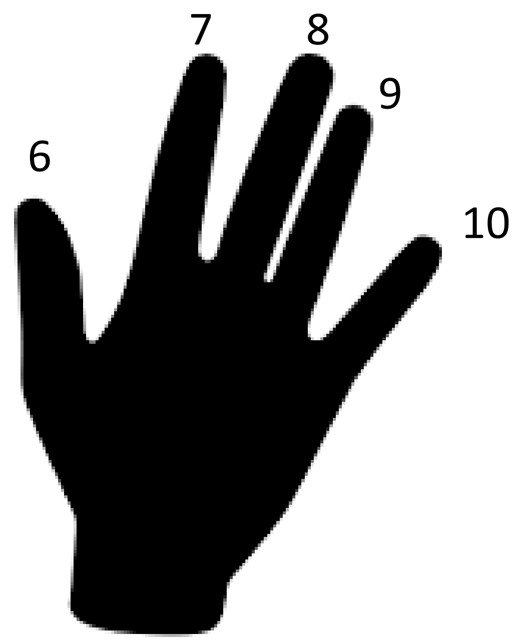


Key Concepts and Critical Aspects

How many fingers on your left hand?



How many fingers on your right hand?



Research

$$\frac{3}{105}$$

6- to 7- year olds thought they had ten fingers on their right hand

Neuman (1987) tried to envisage how the problem appeared to the children and what “five” and “ten” meant to them in context.

Necessary Aspects of Number

Ordinal property – each number refers to a place in order

Cardinal property – “manyness”

Numbers are wholes that can be divided up into parts.

To understand number children need to discern all three aspects of number

Mastery

Mastering an educational objective amounts to **discerning** and taking into consideration its **necessary aspects.**

Misconceptions originate from the fact that we discern some critical aspects but not others.

Shanghai Teaching

Identify necessary/critical aspects of learning

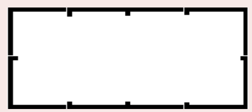
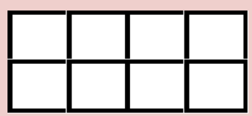
Identify all possible misconceptions

Collaboratively plan and analyse sequences of lessons around key concepts and misconceptions

Teaching Sequences

Perimeter first, followed by area

Instructional design of the topic on perimeter and area of rectangles and squares

sequence	occasion	component			Lesson Plan
		concept	law	unit	
Perimeter first	When learn addition	 segment by segment	Addition	One centimeter	Teach students how to learn Help students to link addition with length
Followed by area	When learn multiplication	 cell by cell	Multiplication	One square centimeter “like the size of the thumb nail”	Advise student to learn by using the already-known method (transfer) Link between multiplication with area

Note: Disconnection between numbers and figures may impede a full and through understanding. Three big ideas in primary mathematics—— conception of numbers, number operations, connection between numbers and figures

Variation Theory

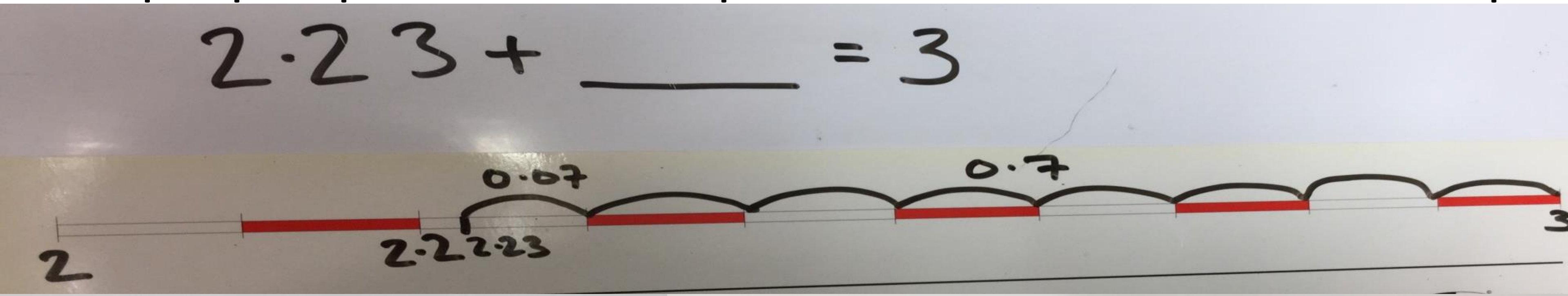
Conceptual variation

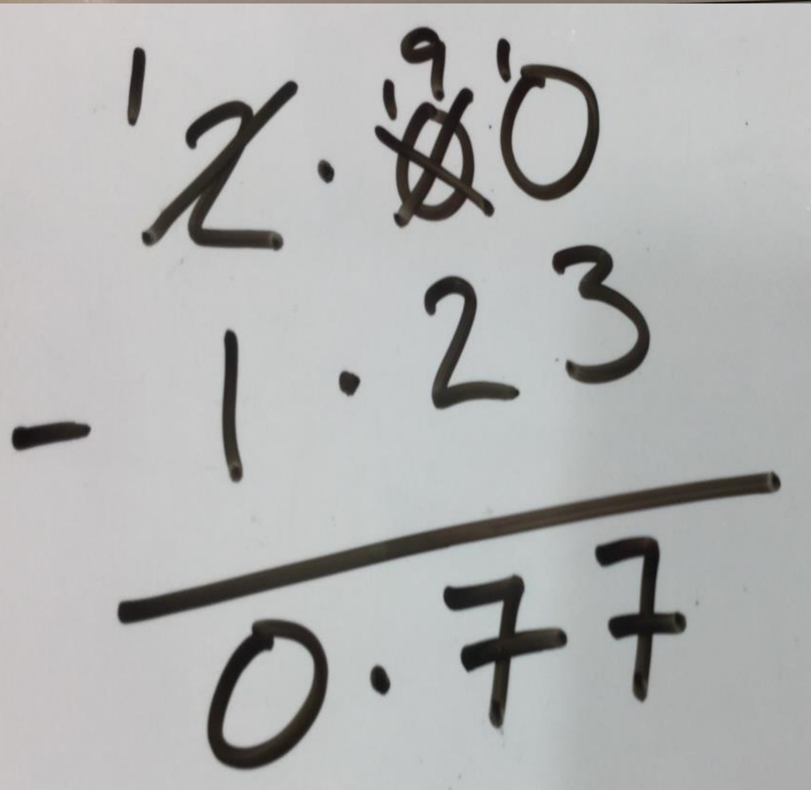
Concept – non concept

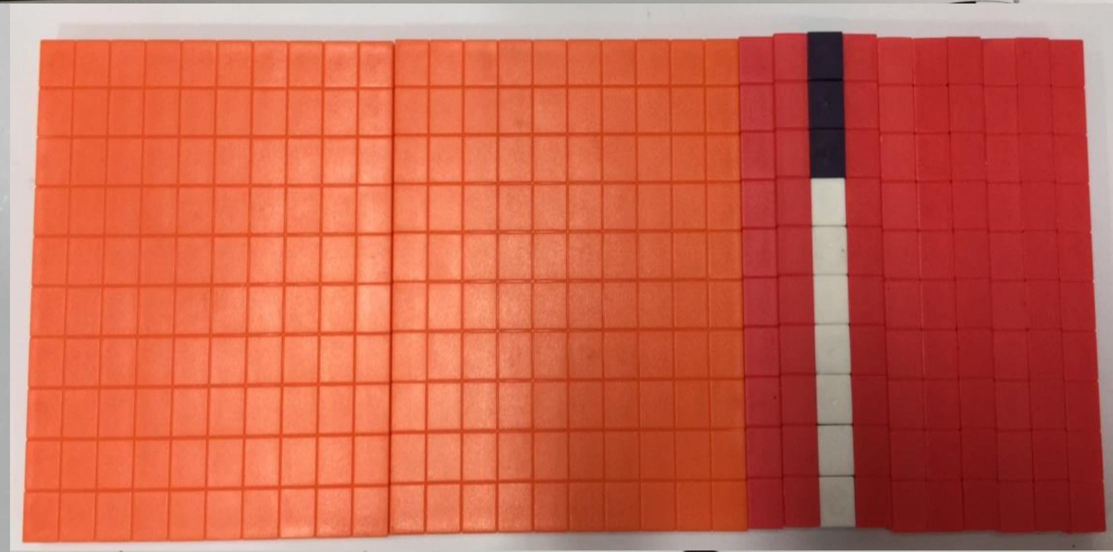
Procedural variation

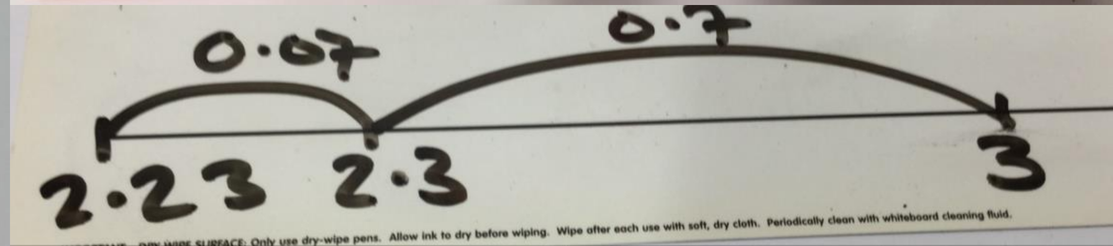
Conceptual Variation

Multiple perspectives and experiences of mathematical concepts



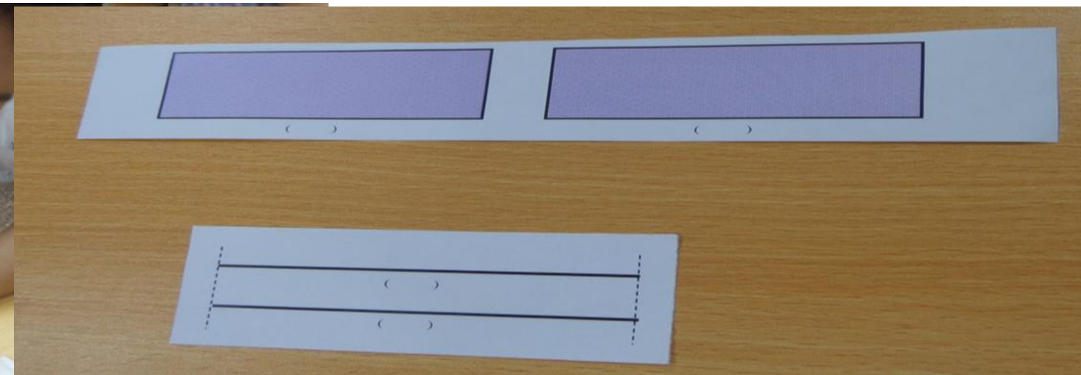




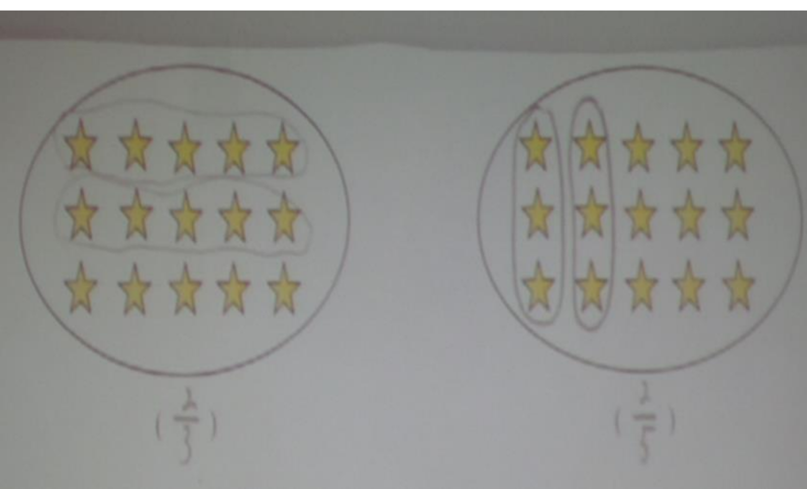


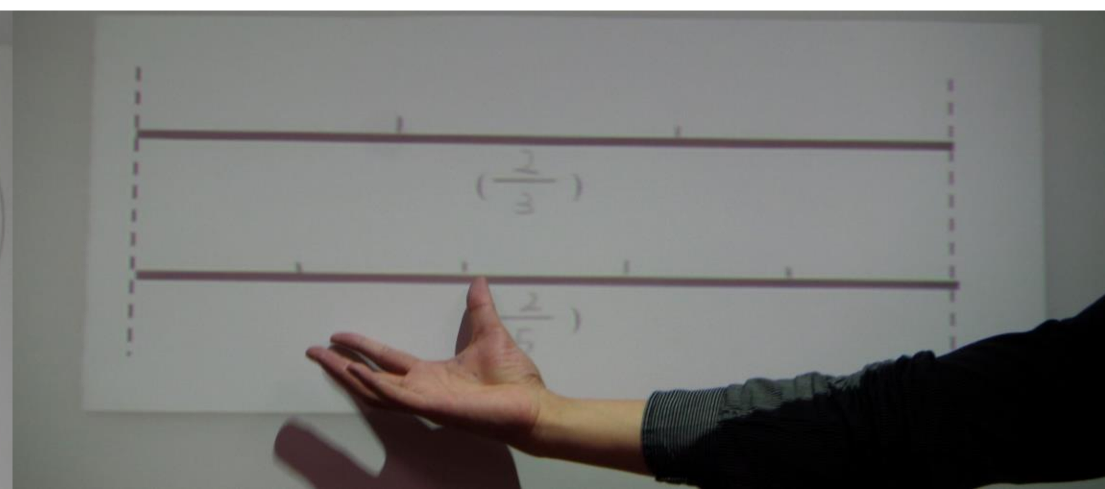
Conceptual Variation

Multiple perspectives and experiences of mathematical concepts



2/5 and 1/3





Conceptual Variation

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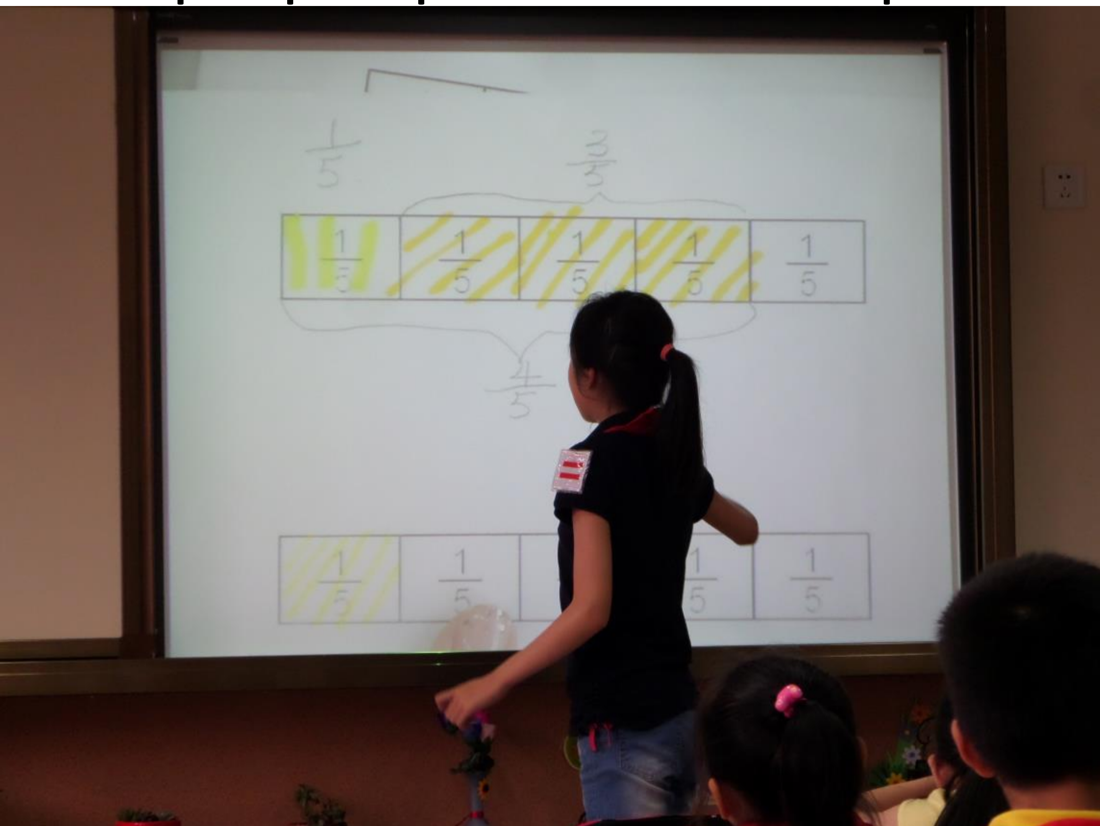
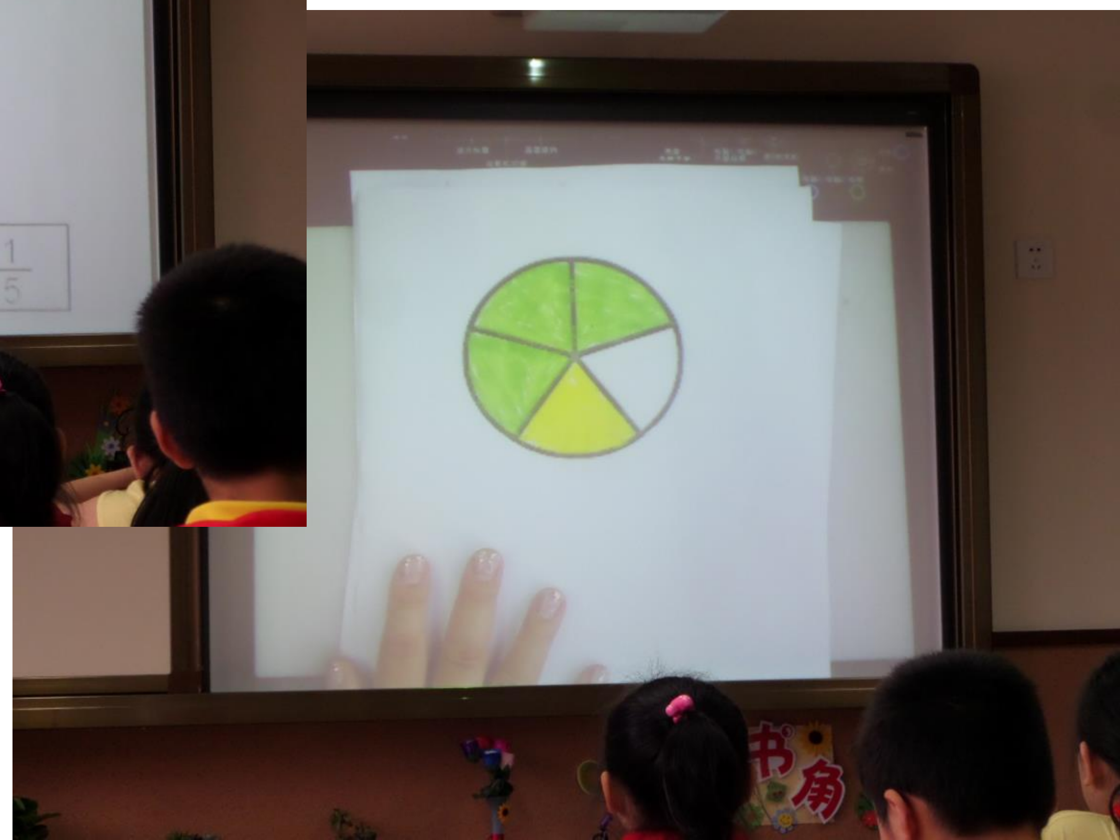


3/5 + 1/5



3/5 – 1/5

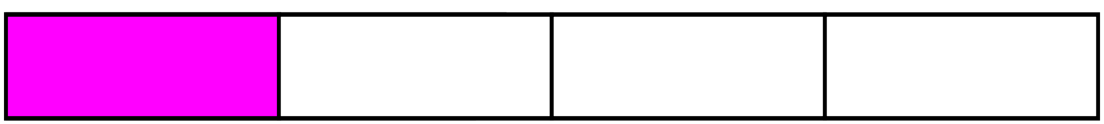
Looking at all aspects of the concept

Tasks which challenge and provoke reasoning□

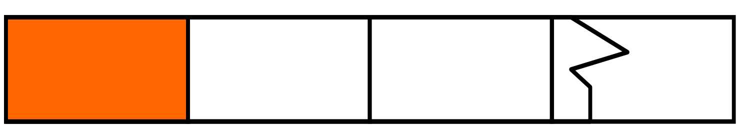
2 paper tapes were broken, can you guess which original paper tape is longer?

Why? How do you get your answer?□

$\frac{1}{5}$

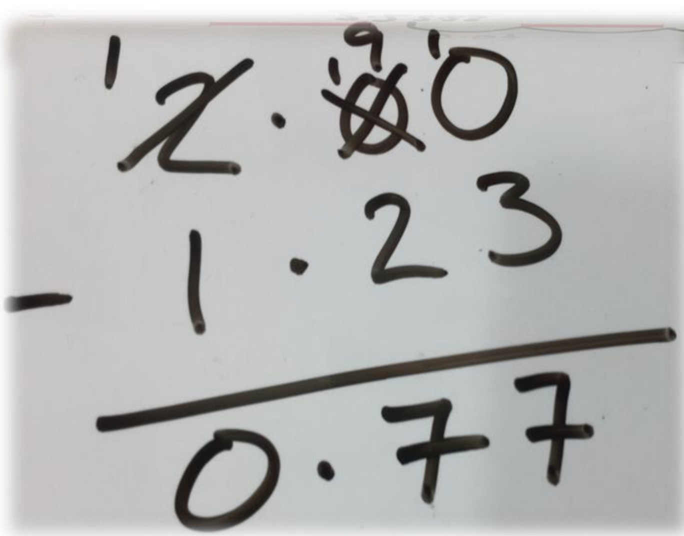


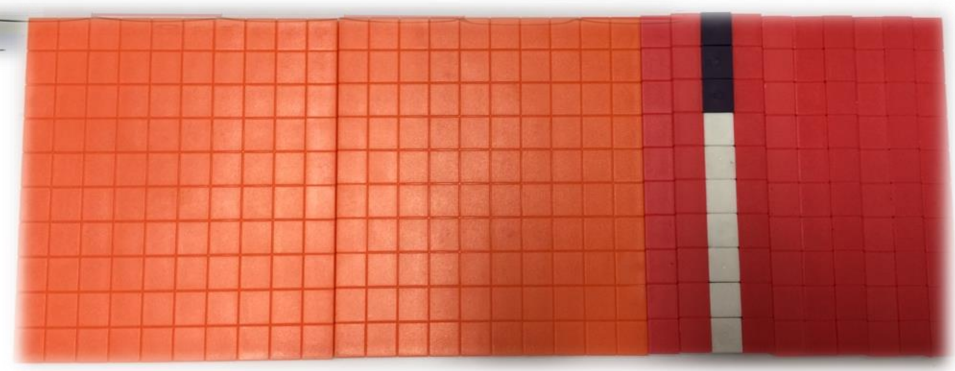
$\frac{1}{5}$



Conceptual variation

the different representations of the concept throughout the lesson or a series of lessons





Concept and Non-Concept

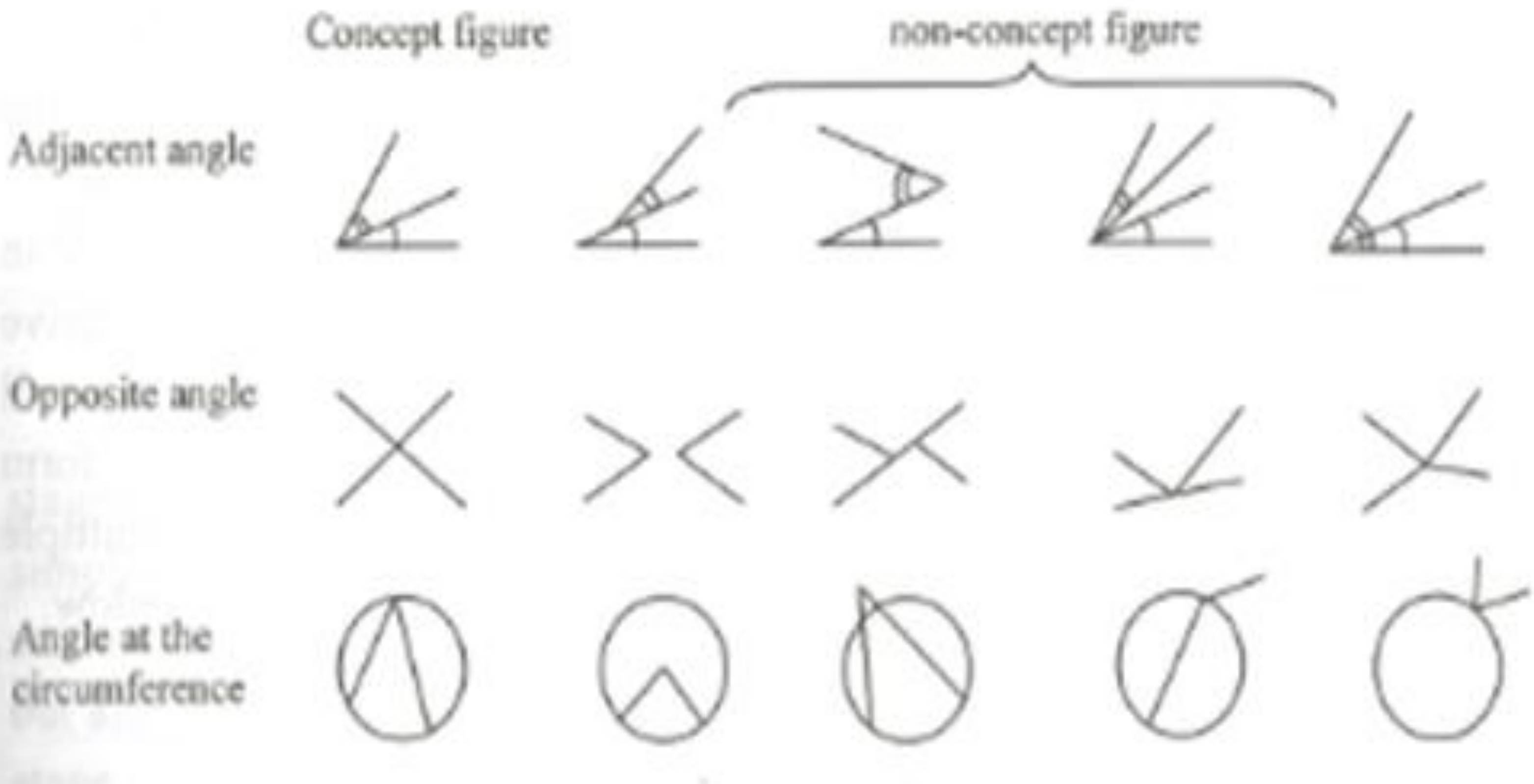
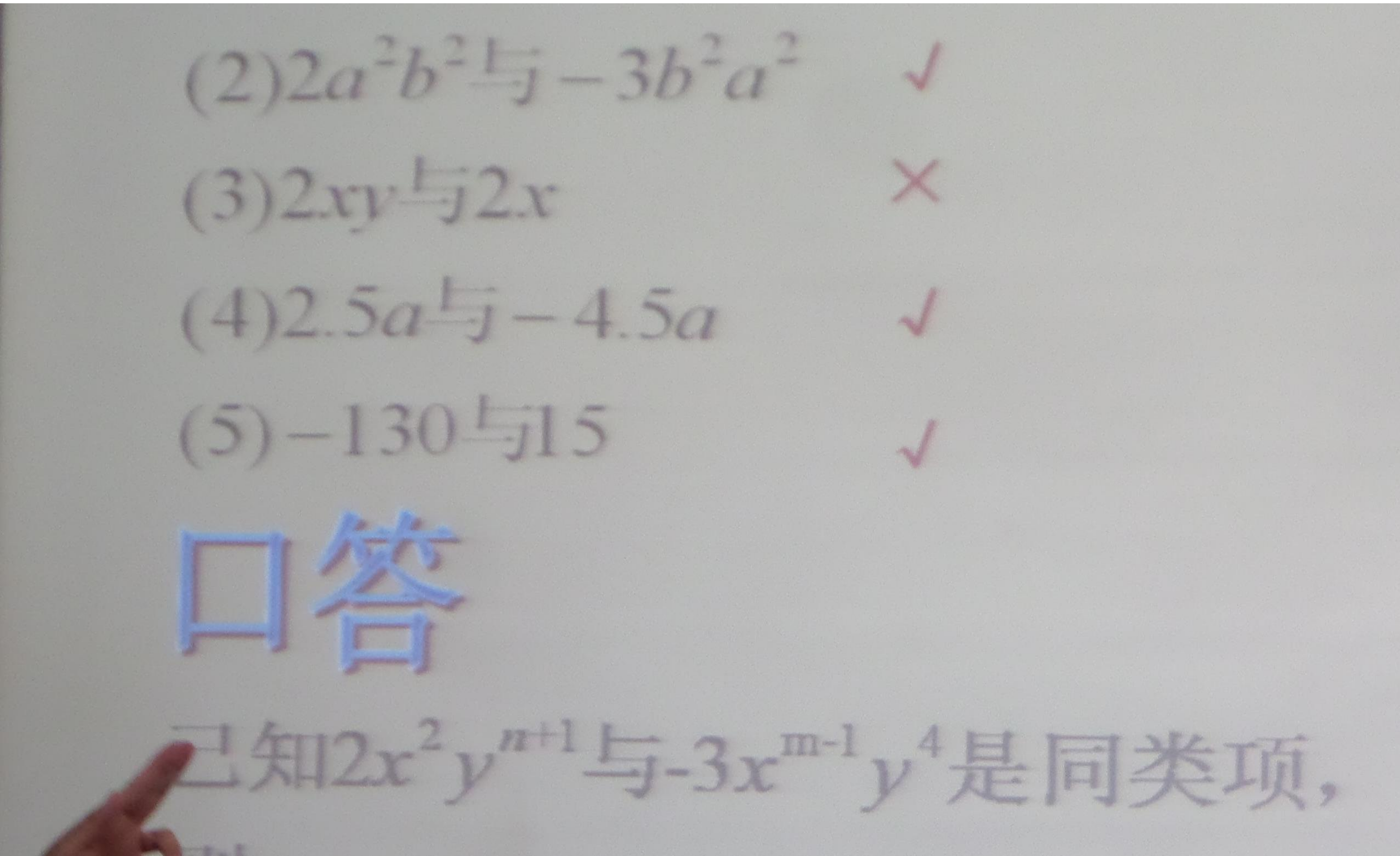


Figure 4. Non-concept figure variations for discerning the essence of concepts (L. Gu, 1981)

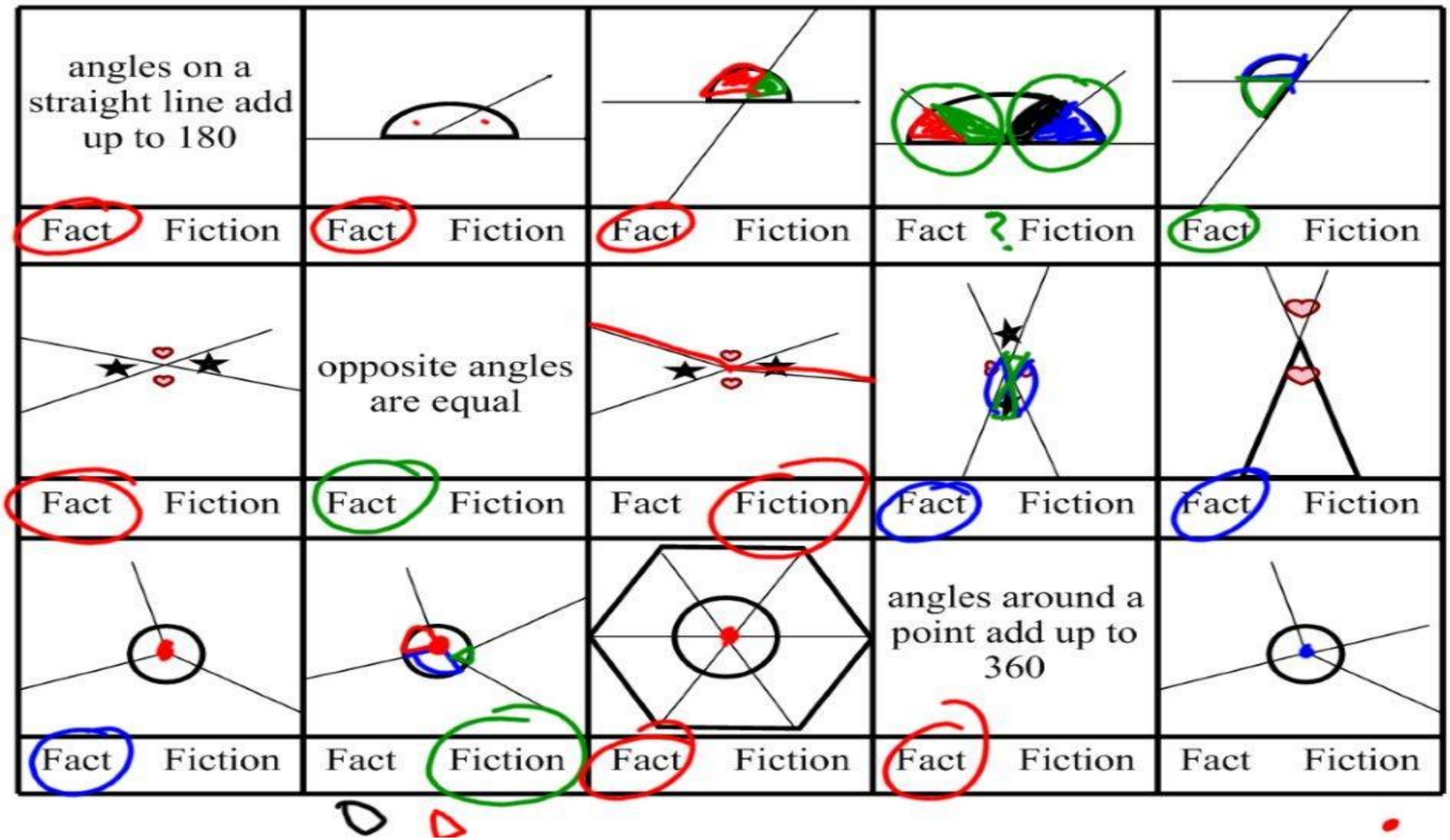
True/false activities

Concept and Non-concept

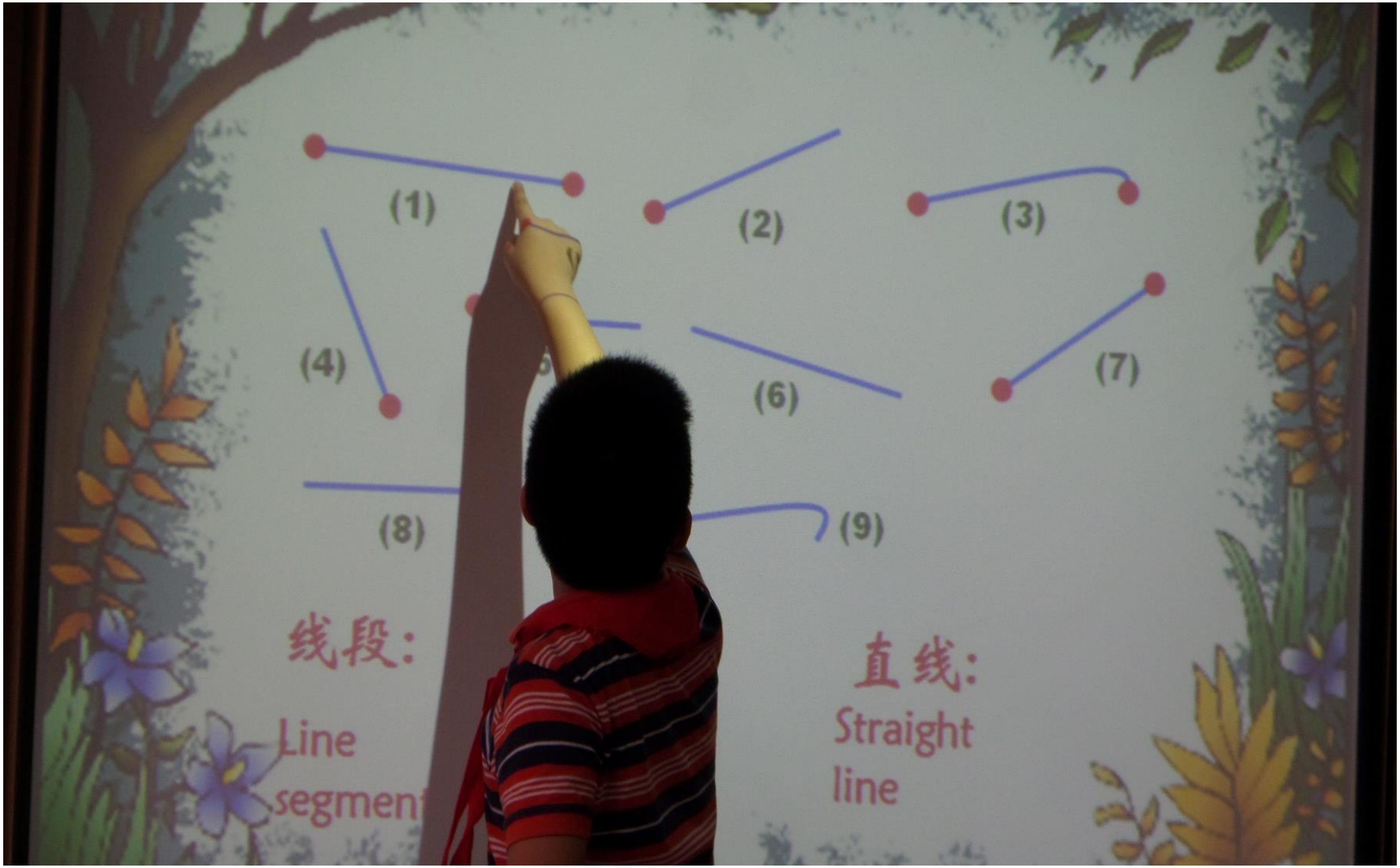


Concept and Non-concept

Angle Facts and Angle Fiction



Concept and Non-concept



Concept and Non-concept

highlighting common misconceptions by looking at the concept and misconcept simultaneously

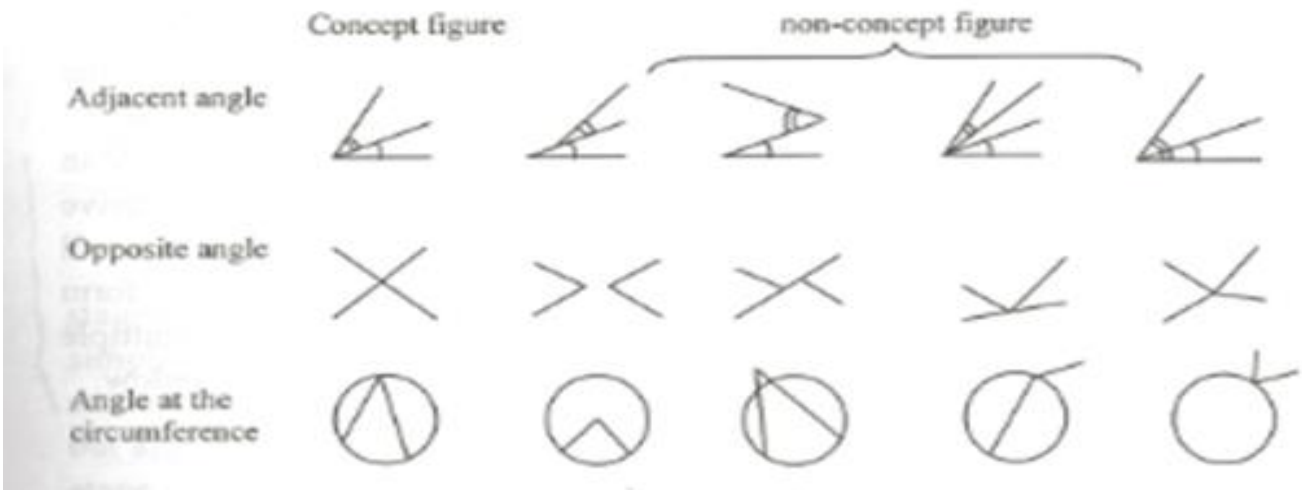


Figure 4. Non-concept figure variations for discerning the essence of concepts (L. Gu, 1981)

True/false activities

Procedural Variation

Progressively unfolding mathematics activities

We tend to associate procedural learning with rote Learning.

However it includes:

- step by step connections which enhance the formation of concepts
- Multiple approaches – to support conceptual deepening

Procedural Variation

(Still about the conceptual)

$2 \times 3 =$	$6 \times 7 =$	$9 \times 8 =$
$2 \times 30 =$	$6 \times 70 =$	$9 \times 80 =$
$2 \times 300 =$	$6 \times 700 =$	$9 \times 800 =$
$20 \times 3 =$	$60 \times 7 =$	$90 \times 8 =$
$200 \times 3 =$	$600 \times 7 =$	$900 \times 8 =$

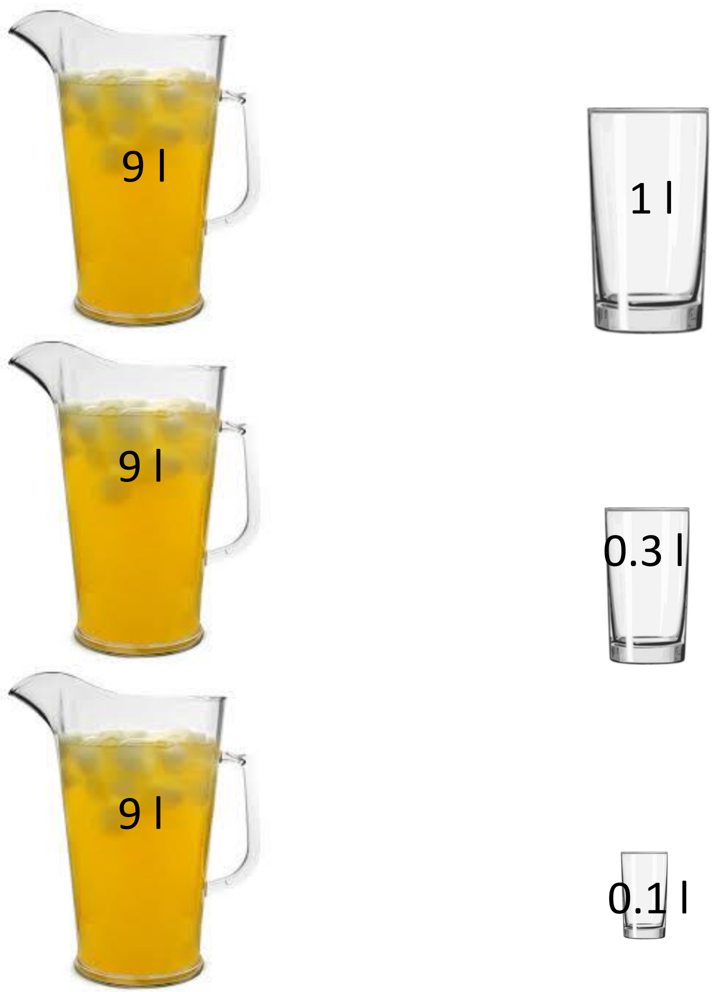
The pupil is carrying out the procedural operation of multiplication, but through connected calculations (variance and invariance):

- Is able to use one calculation to work out another - making reasoned connections between the previous question and the new question.
- has the opportunity to think about key concepts involving multiplication and place value

This leads to intelligent practice

Procedural Variation – Varying the Problem

Consolidate a concept by varying the conditions, changing the results and making generalisations

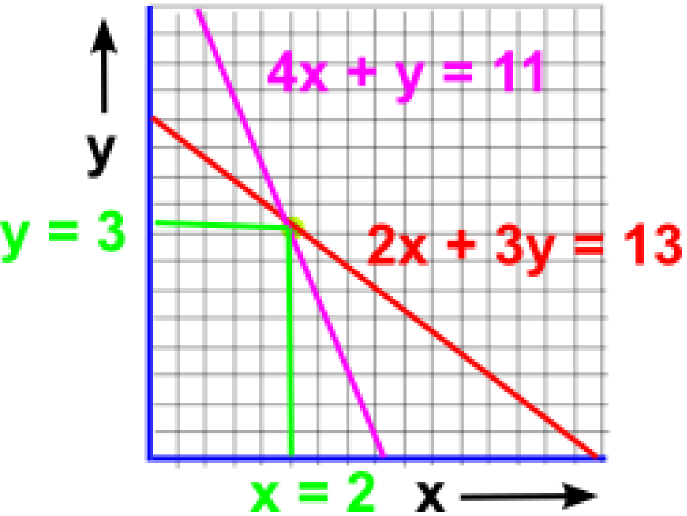


invariant

variant

How many glasses can you fill from a 9 litre jug?

Procedural Variation – Multiple Methods of Solving the Problem



graphical

4x + y =11 (1)
2x + 3y = 13 (2)

12x + 3y = 33 (1)x3
2x + 3y = 13 (2)
10x = 20
x = 2

elimination

2x + 3(11 – 4x) = 13
2x + 33 – 12x = 13
-10x + 33 = 13
20 = 10x
2 = x

substitution

Teach all three together and connect them.

Teach them separately and students will reject two

Procedural Variation

What stays the same?

What's different?

invariant

variant

Basic Units

Represent $5 \div 4 = \frac{5}{4}$

English Sharing Pizza

Share 5 pizzas between 4 people

1 remainder 1

1 pizza each
with 1 left over

1 ¼

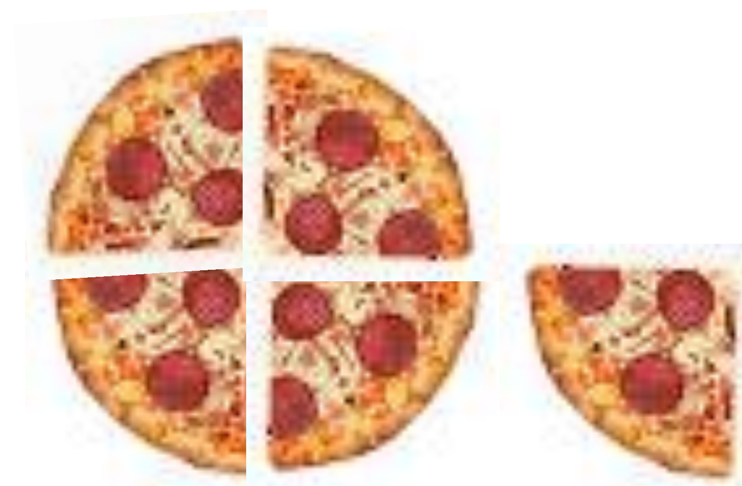
Shanghai Sharing Pizza

Share 5 pizzas between 4 people

$5 \div 4 = 5$ lots of $\frac{1}{4}$

$\frac{5}{4}$

$1\frac{1}{4}$



Teaching for mastery in Shanghai

Think carefully about

- the best **real-world (concrete) representation / model** to introduce the **(abstract) concept**
- the **mathematical reasoning** and **discussions** that should take place in lessons
- the **misconceptions** the pupils **will** have, and how these can be cleared up
- **intelligent practice** (questions and problems) to give **fluency** and **deep understanding**
- the **connections** the pupils need to make

What’s the same, What’s different?

Characteristics of Chinese and British mathematics textbooks