MK MASTERY

Milton Keynes Academy

Sweat the small stuff

Everyone Achieves.

The Secrets of Shanghai Mathematics Teaching

Mastering an educational objective amounts to discerning and taking into consideration its necessary aspects.

Misconceptions originate from the fact that we discern some critical aspects but not others.

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Teacher Research Groups

Identify possible misconceptions

Identify necessary/critical aspects of learning

Collaboratively review and adapt sequences of lessons around key concepts and misconceptions

Teachers of the same year group meet weekly to plan expertly crafted lessons around student misconceptions. This time is also used to moderate work and analyse assessments.

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Teaching with Variation

The central idea of teaching with variation is to highlight the essential features of the concepts through varying the non-essential features.

Gu, Huang & Marton, 2004

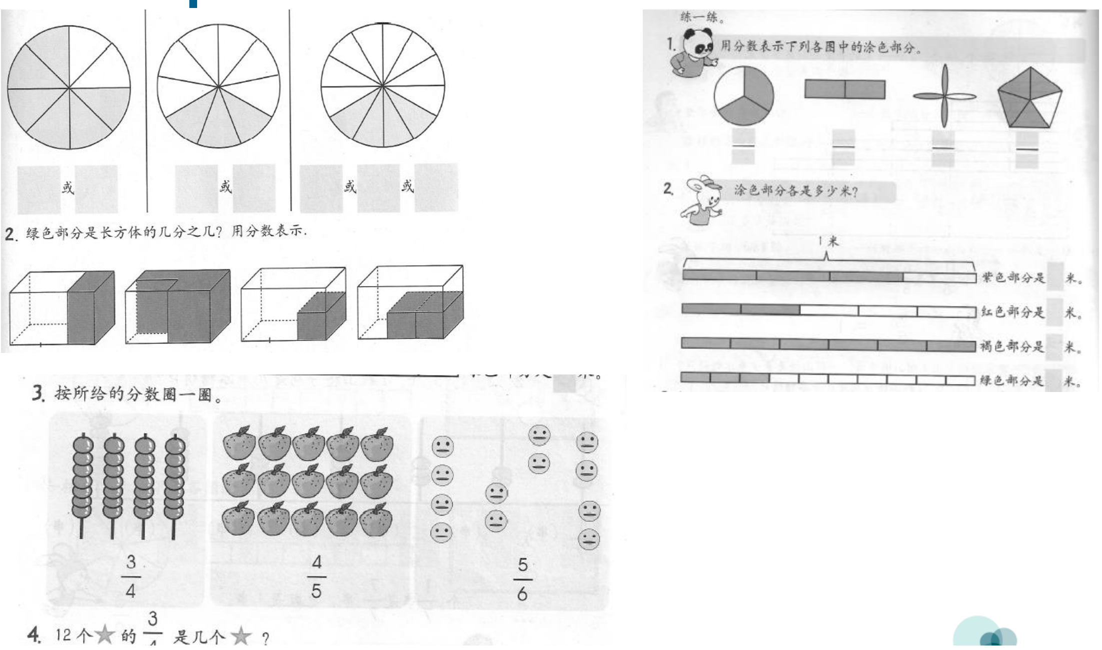
Variation theory is posited on the view that "when certain aspects of a phenomenon vary while its other aspects are kept constant, those aspects that vary are discerned".

Lo, Chik & Pang, 2006

Conceptual Variation

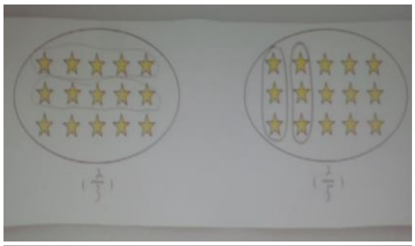
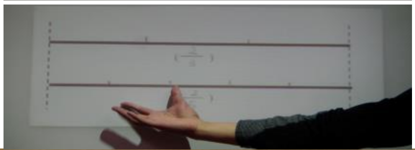
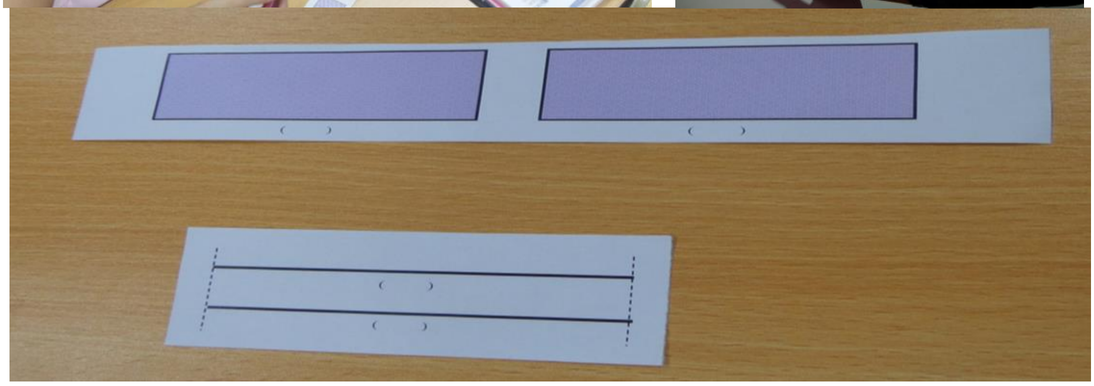
Conceptual variation (see Lo & Marton, 2012; Leung, 2003) aims at providing students with multiple perspectives and experiences of mathematical concepts (Gu, Huang & Marton, 2004).

Conceptual Variation



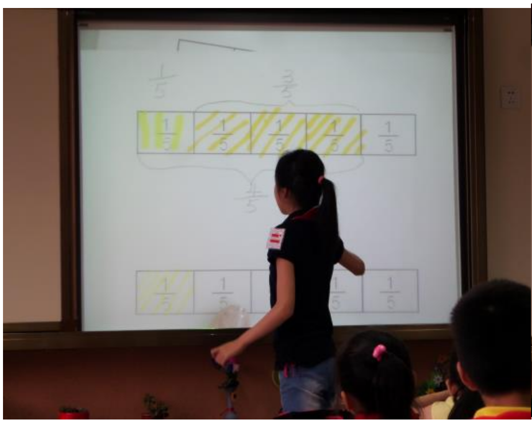
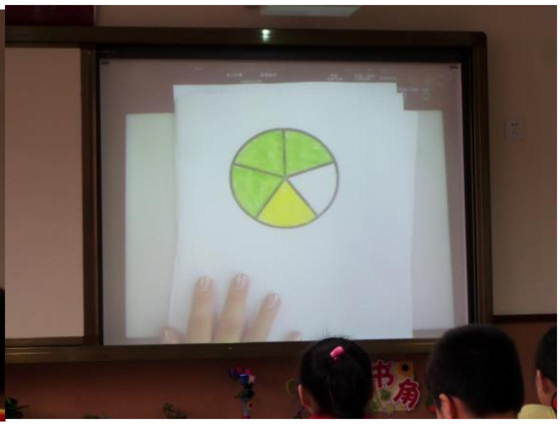
Conceptual Variation – Grade 4 -Shanghai 2015

Represent 2/5 and 2/3

Conceptual Variation – Grade 4 - Shanghai 2015

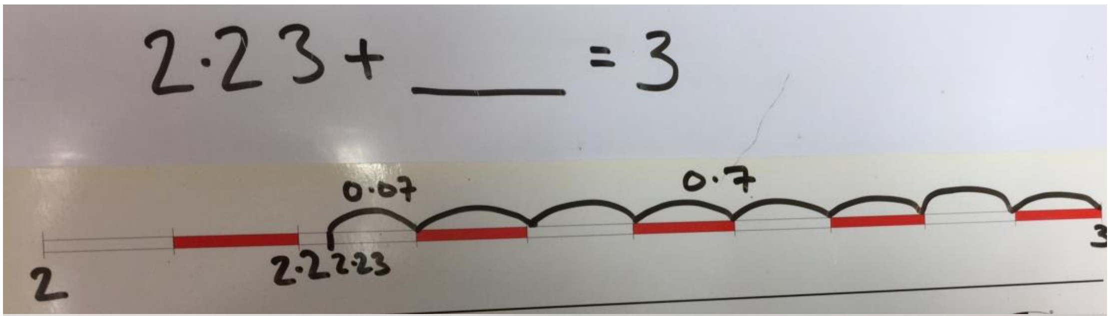
3/5 + 1/5 and 3/5 – 1/5

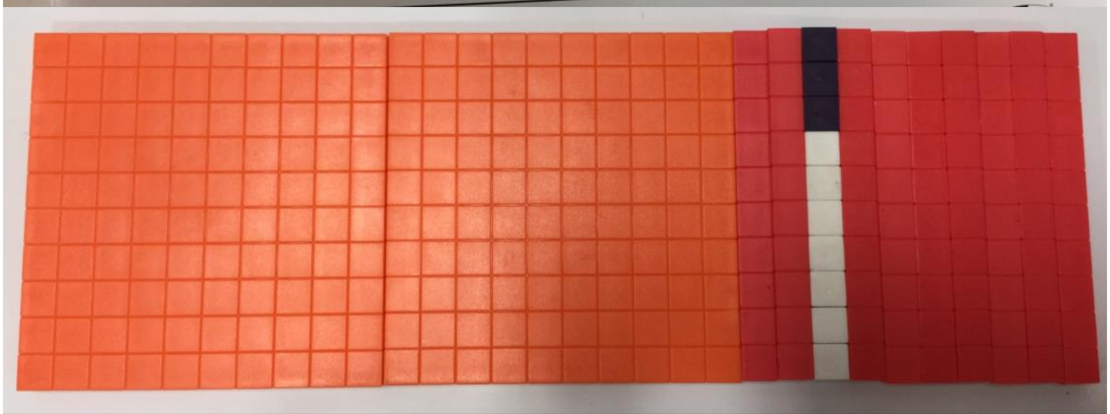



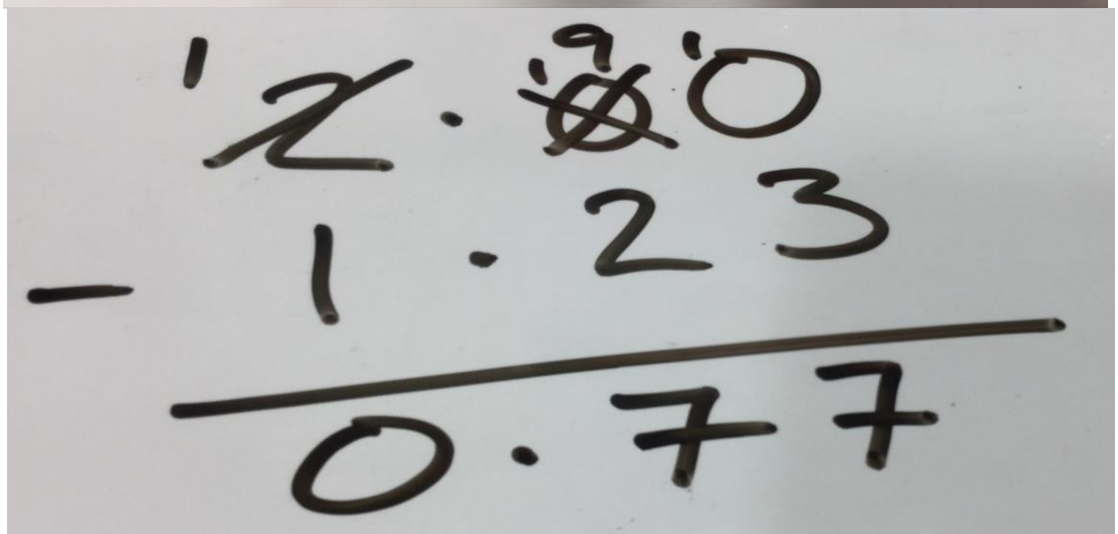
MathsHUBS Enigma

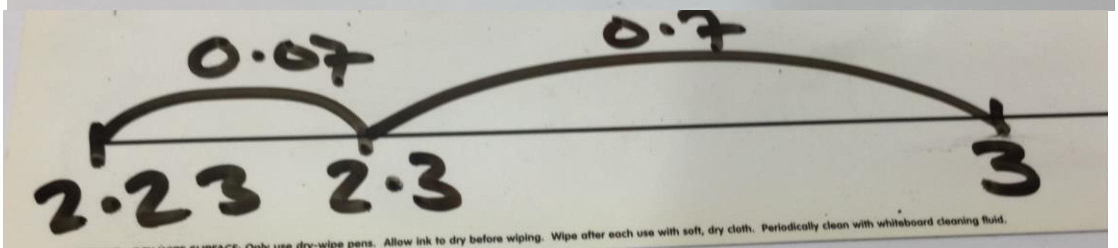
Milton Keynes Academy

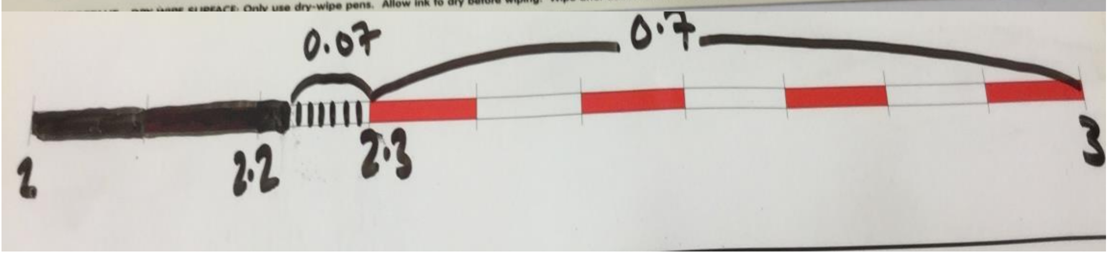
Conceptual variation – Year 7 - MKA 2015











Concept – Non-Concept

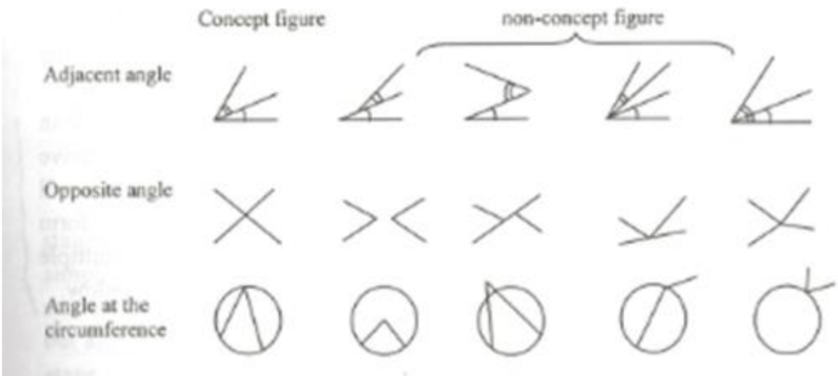
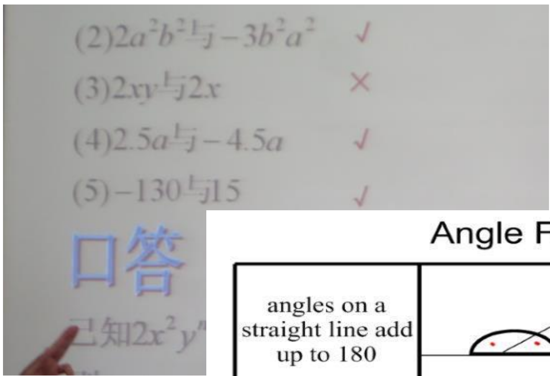
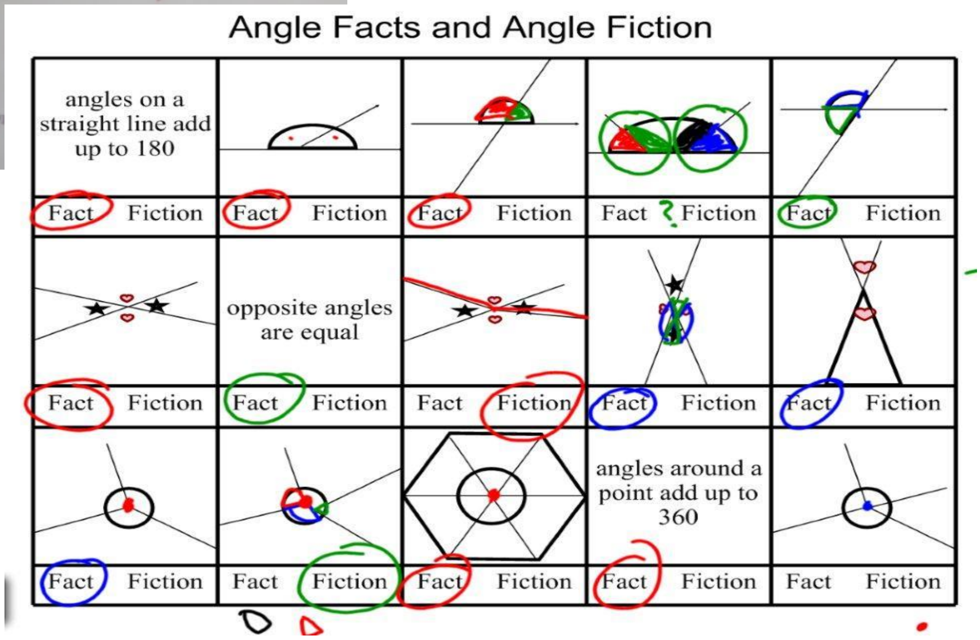


Figure 4. Non-concept figure variations for discerning the essence of concepts (L. Gu, 1981)

Show examples of concept and non-concept simultaneously. This draws out misconceptions at the very start. Students learn what the concept is by comparing to what it is not.

like and unlike terms





Procedural Variation

Progressively unfolding mathematics activities
We tend to associate procedural learning with rote Learning.
However it includes:

- step by step connections which enhance the formation of concepts
- Multiple approaches – to support conceptual deepening

Procedural Variation

(Still about the conceptual)

$2 \times 3 =$	$6 \times 7 =$	$9 \times 8 =$
$2 \times 30 =$	$6 \times 70 =$	$9 \times 80 =$
$2 \times 300 =$	$6 \times 700 =$	$9 \times 800 =$
$20 \times 3 =$	$60 \times 7 =$	$90 \times 8 =$
$200 \times 3 =$	$600 \times 7 =$	$900 \times 8 =$

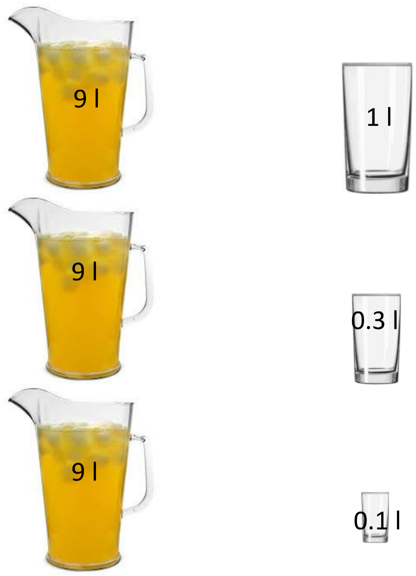
The pupil is carrying out the procedural operation of multiplication, but through connected calculations (variance and invariance):

- Is able to use one calculation to work out another - making reasoned connections between the previous question and the new question.
- has the opportunity to think about key concepts involving multiplication and place value

This leads to intelligent practice

Procedural Variation – Varying the Problem

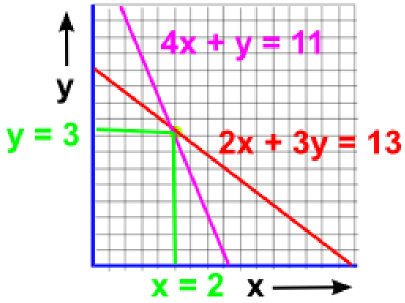
Consolidate a concept by varying the conditions, changing the results and making generalisations



How many glasses can you fill from a 9 litre jug?

invariant variant

Procedural Variation – Multiple Methods of Solving the Problem



graphical

$4x + y = 11 \quad (1)$

$2x + 3y = 13 \quad (2)$

$12x + 3y = 33 \quad (1)\times3$

$\underline{2x + 3y = 13 \quad (2)}$

$10x = 20$

$x = 2$

elimination

$2x + 3(11 - 4x) = 13$

$2x + 33 - 12x = 13$

$-10x + 33 = 13$

$20 = 10x$

$2 = x$

substitution

Teach all three together and connect them.

Teach them separately and students will reject two

Basic Units

We usually define basic units as ones(units)

123 is the same 123 ones

Which often leads to

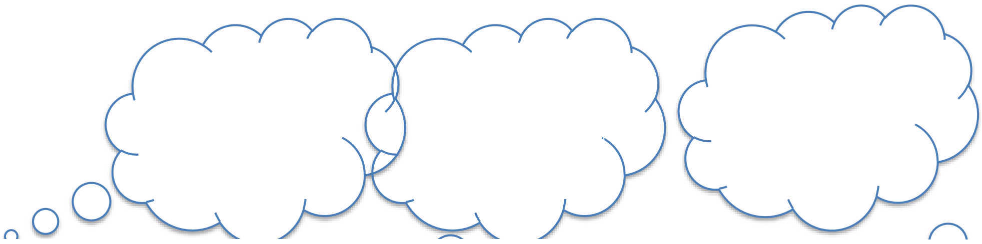
```
  123
× 645
-----
  615
  492
  738
-----
1,845
```

Change the basic units to units, tens and hundreds

```
   123
 × 645
------
   615      123 x 5 units
  492       123 x 4 tens
 738        123 x 6 hundreds
------
79,335
```

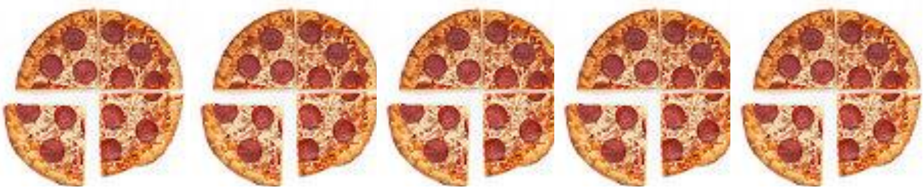
English Sharing Pizza

Share 5 pizzas between 4 people



Shanghai Sharing Pizza

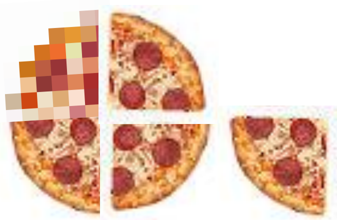
Share 5 pizzas between 4 people



$5 \div 4 = 5$ lots of $\frac{1}{4}$

$\frac{5}{4}$

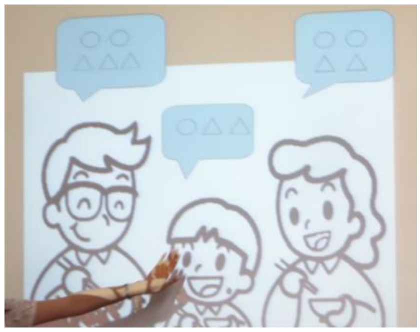
$1\frac{1}{4}$



What's the same, What's different?

MKA Mastery
We have slimmed down the KS3 curriculum to focus on a smaller number of topics in greater depth. We aim to address misconceptions early and deepen understanding. Once students gain fluency they develop reasoning and problem solving skills. This is all underpinned by carefully crafted lessons using Variation Theory. Years 7 and 8 are taught in mixed ability classes with a specialist Maths teacher providing intervention during registration and ERIC time. (Everyone Reads in Class for the first 10 minutes of every lesson in KS3)

Shanghai Style
Most Shanghai Maths lessons start with a real life, simple introduction to the concept and very quickly move into the abstract.



问题

下列各组单项式是不是同类项?

(1) $3x^2y$ 与 $2y^2x$;

(2) $2a^2b^2$ 与 $-3b^2a^2$;

(3) $2xy$ 与 $2x$;

(4) $2.3a$ 与 $-4.5a$.

Starting with a very basic entry point and moving quickly into the abstract proved to be popular with mixed ability classes in year 7 and 8.

"I like the fact that we are all doing the same thing" – year 7

Differentiation and Challenge
Every lesson is accessible to all students. Varying the concept provides support and consolidation for the least able builing resilience and confidence. Addressing misconceptions and insisting on the use of correct language often shows up a lack of deep understanding and reliance on procedure in the most able. Encouraging **all** to regularly generalise using bar model and algebra deepens the understanding of all students and provides sufficient challenge for the most able.

MKA Mastery Booklets and Notebooks
In year 7 – 11 student booklets and notebooks are designed using Variation Theory. Each lesson addresses one very narrow key concept to be mastered. Mastery of the concept is assessed using exit tickets at the end of each lesson and recorded on a central tracking sheet at the end of each topic. Resources are reviewed and adapted weekly in Teacher Research Groups. Intervention is immediate or before the next lesson using feedback, adapting future lessons and specialist maths teacher intervention.

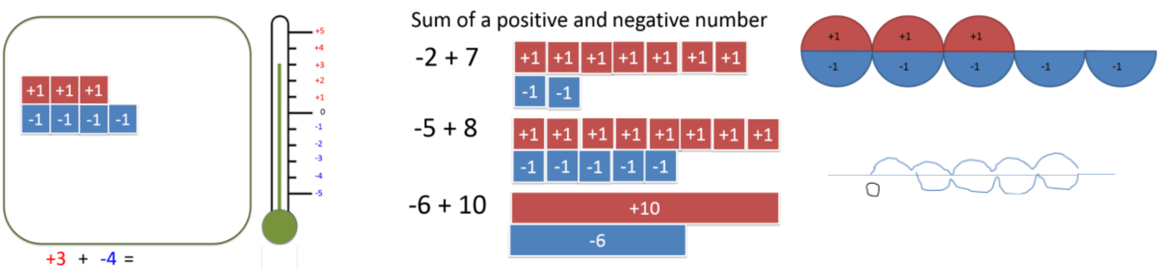
Planning a Mastery lesson

Here are two examples of planning one lesson out of a unit of work. Each lesson has one narrow concept in greater depth.

Negative Numbers – Year 7

Key concept - Positive + negative

Conceptual variation



Intelligent Practice (procedural variation)

3+ 1 =
3+ 0 =
3+ -1 =
3+ -2 =
3 +-3 =
3+ -3 =
3+ -4 =
3+ -25 =
3+ -127 =

Avoid students answering questions by spotting patterns.

Generalise

Move quickly to questions that rely on a deeper understanding of the structure of the problem

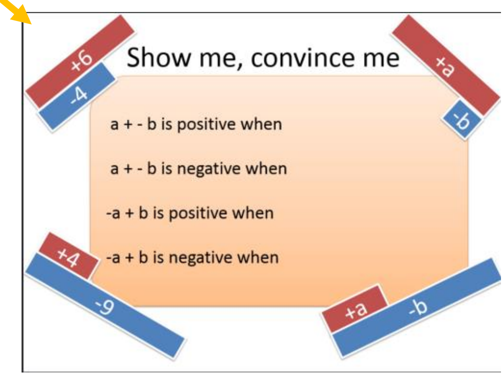
3+ -a >0 when
3+ -a = 0 when
3+ -a < = when

Investigate

a + -b >0 Write three questions that make this statement true

a + -b =0 Write three questions that make this statement true

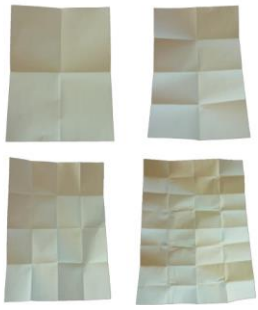
a + -b <0 Write three questions that make this statement true



Indices – Year 10

Key concept – addition law of indices

Introduction of concept
Paper folding - https://www.scribd.com/doc/53058897/Index-Laws-Folding-Paper-v1



Gremlins – A gremlin lives for one hour then splits in two.

t = 0   1 gremlin

t = 1 2 gremlins

t = 2 4 gremlins

How many gremlins after 5 hours? How long would it take before gremlins took over the school?

Concept – Non concept review of prior learning

True or False
$2^4 = 8$
$2^{10} = 20$
$2^{-1} = -2$
$3^4 = 12$
$2^4 = 16$
$-1^4 = -1$
$-1^4 = 1$

Addressing common misconception $a^m = m \times a$ at the start

Provide challenging questions for a richer discussion

Indices practice
$2x2x2x2 = 2^n$
$3x3x3x3x3x3x3 = 3^m$
$5x\ 5x5 = m^n$
$2x2x2x2x2x5x5x5x5 = 2^m \times 5^n$

Addition law of indices

Example

$2^2 \times 2^3 = 2\times2 \times 2\times2\times2 = 2^5$

Intelligent Practice (procedural variation)

$2^2 \times 2^4$

$2^2 \times 2^7$

$2^2 \times 3^8 \neq 6^{10}$ explain why this statement is true

$2^2 \times 4^5 = 2^n$ n = ...

$2^2 \times 8^5$

$2^2 \times m^3 = 2^{17}$ m=.........

$2^2 \times 6^5 = 2^m \times 3^n$ m= ...... n=........

Move quickly to challenging questions that rely on a deeper understanding of the structure of the problem.

This avoids 10 ticks syndrome – 10 easy questions completed in super quick time with little thought leading to students false elief that they have understood the concept! A very common

True or False (address misconceptions)

$2^2 + 2^3 = 2^5$

$2^2 \times 2^3 = 2^6$

$2^m \times 2^n = 2^{mn}$

$2^m \times 2^n = 2^{m+n}$

$2^m \times 2^n = 4^{m+n}$

$2^m \times 2^n = 4^{mn}$

Challenge

$2^{m+1} \times 2^{m-1} = 2^7$

Thinking Task

Show me which of these are square numbers

$2^2 \times 3^1$

$2^2 \times 3^2$

$2^2 \times 3^4$

$2^2 \times 3^5$

$2^2 \times 3^6$

Which values of n make this expression $2^2 \times 3^n$ a square number?

Which values of m and n make this expression $2^m \times 3^n$ a square number?

Convince me that $a^{2m} \times b^{2n}$ is a square number and find its square root.

LESS IS MORE

Narrow the concept and deepen understanding.

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